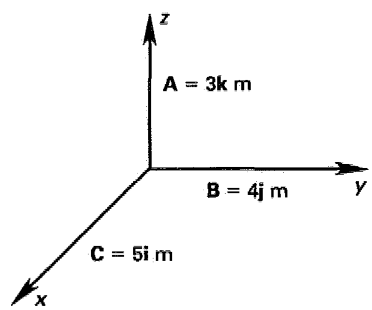
8 STATICS

STATICS–1

What is the length of the vector $\mathbf{A}+\mathbf{B}+\mathbf{C}$, the sum of three orthogonal vectors?



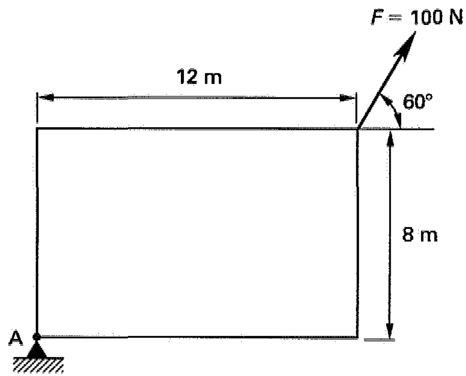
(A) 3.5 m (B) 4.3 m (C) 7.1 m (D) 10 m

$$
\begin{aligned}
|\mathbf{A}+\mathbf{B}+\mathbf{C}| &= \sqrt{A^2+B^2+C^2} \\
&= \sqrt{(3\text{ m})^2+(4\text{ m})^2+(5\text{ m})^2} \\
&= 7.07\text{ m} \quad (7.1\text{ m})
\end{aligned}
$$

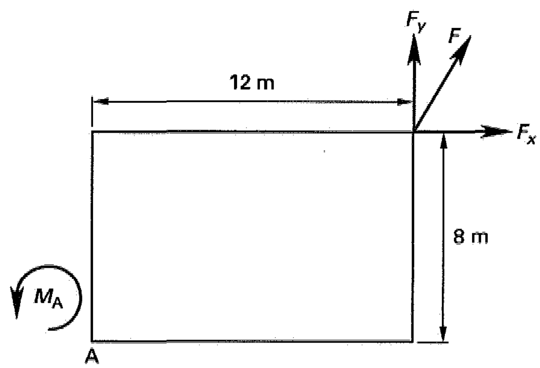
The answer is (C).

STATICS–2

Determine the magnitude of the moment of the force F about the corner A.



(A) 120 N·m (B) 240 N·m (C) 320 N·m (D) 640 N·m



$$F_x = (100 \text{ N})\cos 60° = 50.0 \text{ N}$$
$$F_y = (100 \text{ N})\sin 60° = 86.6 \text{ N}$$

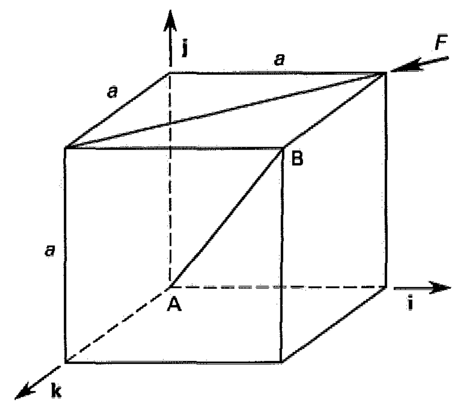
Taking counterclockwise moments as positive,

$$\sum M_A = -yF_x + xF_y$$
$$= -(8 \text{ m})(50.0 \text{ N}) + (12 \text{ m})(86.6 \text{ N})$$
$$= 640 \text{ N·m}$$

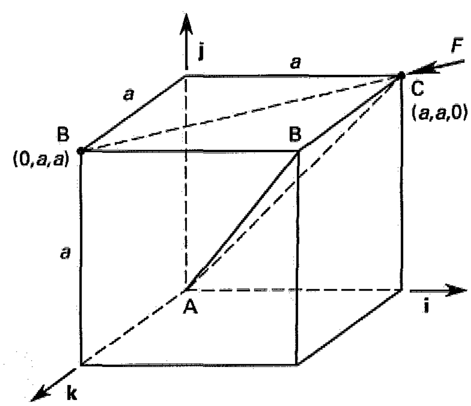
The answer is (D).

STATICS–3

A cube of side length a is acted upon by a force F as shown. Determine the magnitude of the moment of F about the diagonal AB.



(A) $\frac{aF}{\sqrt{8}}$ (B) $\frac{aF}{\sqrt{6}}$ (C) $\frac{aF}{\sqrt{4}}$ (D) $\frac{aF}{\sqrt{3}}$



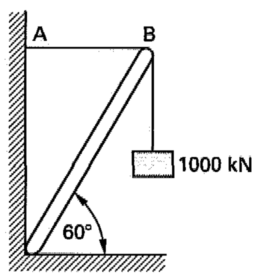
$$M_A = \mathbf{r}_{AC} \times \mathbf{F}$$
$$= a(\mathbf{i}+\mathbf{j}) \times \frac{F}{\sqrt{2}}(-\mathbf{i}+\mathbf{k})$$
$$= \frac{aF}{\sqrt{2}}(\mathbf{i}-\mathbf{j}+\mathbf{k})$$

$$\begin{aligned} U_{AB} &= \frac{1}{\sqrt{3}}(\mathbf{i}+\mathbf{j}+\mathbf{k}) \\ M_{AB} &= U_{AB}\cdot M_A \\ &= \left(\frac{1}{\sqrt{3}}(\mathbf{i}+\mathbf{j}+\mathbf{k})\right)\cdot\left(\frac{aF}{\sqrt{2}}(\mathbf{i}+\mathbf{j}+\mathbf{k})\right) \\ &= \frac{aF}{\sqrt{6}} \end{aligned}$$

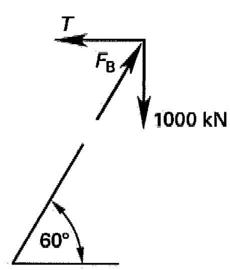
The answer is (B).

STATICS–4

The boom shown has negligible weight, but it has sufficient strength to support the 1000 kN load without buckling. What is most nearly the tension in the supporting cable between points A and B?



(A) 200 kN (B) 430 kN (C) 580 kN (D) 870 kN



For equilibrium, $\sum F_x = 0$ and $\sum F_y = 0$.

$$\sum F_y = 0 = F_B \sin 60° - 1000 \text{ kN} = 0$$

$$F_B = 1154.7 \text{ kN}$$

$$\sum F_x = 0$$

$$F_B \cos 60° - T = 0$$

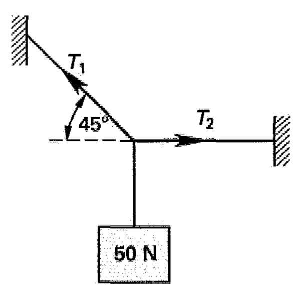
The tension in the cable, T, is

$$\begin{aligned} T &= F_B \cos 60° \\ &= (1154.7 \text{ kN}) \cos 60° \\ &= 577.4 \text{ kN} \quad (580 \text{ kN}) \end{aligned}$$

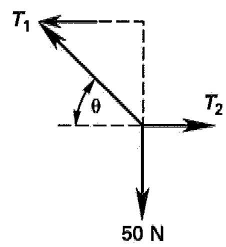
The answer is (C).

STATICS–5

Find the tensions, T_1 and T_2, in the ropes shown so that the system is in equilibrium.



(A) $T_1 = 50.0$ N, $T_2 = 0.0$ N
(B) $T_1 = 50.0$ N, $T_2 = 50.0$ N
(C) $T_1 = 70.7$ N, $T_2 = 50.0$ N
(D) $T_1 = 70.7$ N, $T_2 = 70.7$ N

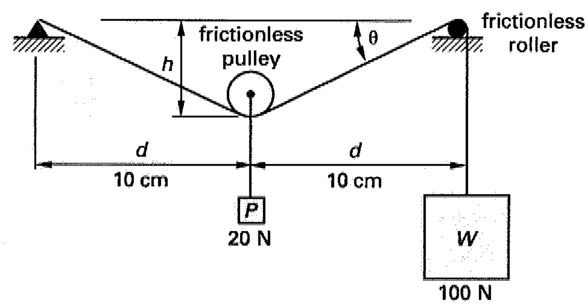


$$\sum F_y = 0 = T_1 \sin 45° - 50 \text{ N} = 0$$
$$T_1 \sin 45° = 50 \text{ N}$$
$$T_1 = 70.7 \text{ N}$$
$$\sum F_x = 0$$
$$T_1 \cos 45° - T_2 = 0$$
$$T_2 = T_1 \cos 45°$$
$$= 50 \text{ N}$$

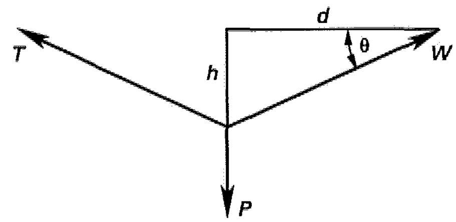
The answer is (C).

STATICS–6

For the system illustrated, determine the value of h that puts the system in equilibrium.



(A) 0.50 cm (B) 1.0 cm (C) 1.5 cm (D) 2.1 cm



$$\sum F_y = 0$$

$$0 = -P + W\sin\theta + T\sin\theta$$

$$(T+W)\sin\theta = P$$

$$\sum F_x = 0$$

$$0 = W\cos\theta - T\cos\theta$$

$$T = W$$

$$= 100\ \text{N}$$

$$2W\sin\theta = P$$

$$\sin\theta = \frac{P}{2W}$$

$$\frac{h}{\sqrt{h^2+d^2}} = \frac{P}{2W} = \frac{20\ \text{N}}{(2)(100\ \text{N})}$$

$$= 0.1$$

$$h^2 = 0.01(h^2+d^2)$$

$$0.99h^2 = 0.01d^2$$

$$h = d\sqrt{\frac{0.01}{0.99}}$$

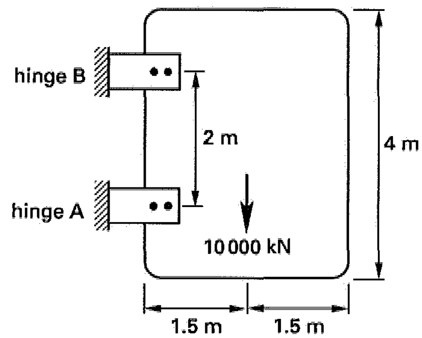
$$= 10\ \text{cm}\sqrt{\frac{0.01}{0.99}}$$

$$= 1.0\ \text{cm}$$

The answer is (B).

STATICS–7

Hinges A and B support a 10 000 kN bank vault door as shown. Determine the horizontal force in the hinge pin at A.



(A) 1500 kN (B) 2500 kN (C) 5500 kN (D) 7500 kN

Sum the moments around joint B.

$$\sum M_B = 0$$

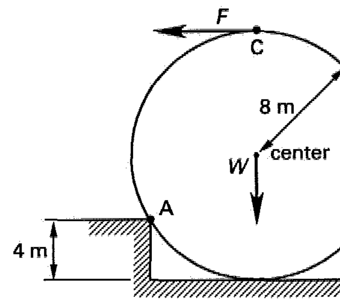
$$0 = (-10\,000 \text{ kN})(1.5 \text{ m}) + (2 \text{ m})R_{Ax}$$

$$R_{Ax} = 7500 \text{ kN}$$

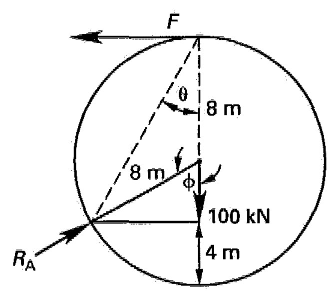
The answer is (D).

STATICS–8

A cylindrical tank is at rest as shown. The tank has a weight of 100 kN. Approximately what horizontal force should be applied horizontally at point C to raise the tank?



(A) 25 kN (B) 58 kN (C) 67 kN (D) 110 kN



In order to raise the tank, $\sum M_A \geq 0$. Therefore, the minimum force that must be applied at point C can be found as follows.

$$\sum M_A = 0$$

$$F(12 \text{ m}) - W(8 \text{ m})\sin\phi = 0$$

$$= \frac{2W\sin\phi}{3}$$

$$\cos\phi = \frac{4 \text{ m}}{8 \text{ m}} = 0.5$$

$$\phi = 60°$$

$$F = \frac{2W\sin 60°}{3}$$

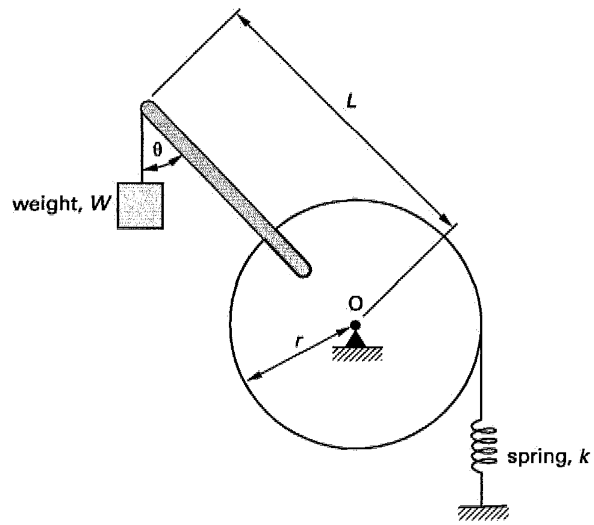
$$= \frac{(2)(100 \text{ kN})\sin 60°}{3}$$

$$= 57.7 \text{ kN} \quad (58 \text{ kN})$$

The answer is (B).

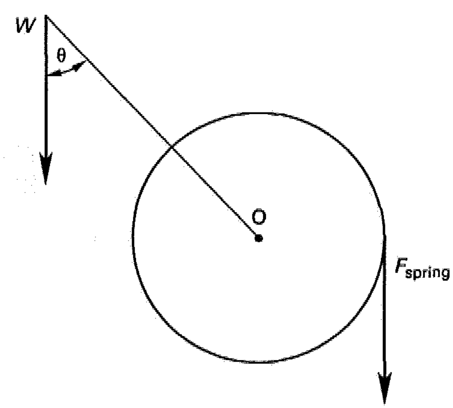
STATICS–9

A weight is attached to a lever as shown. Determine the expression for θ when the system is at equilibrium. The spring constant is k, the length of the lever is L, the radius of the wheel is r, and the magnitude of the weight is W.



(A) $\theta = \frac{WL\sin\theta}{kr}$ (B) $\theta = \frac{WL\sin\theta}{kr^2}$

(C) $\theta = \frac{Wr}{kL}$ (D) $\theta = \frac{WL\cos\theta}{kr}$



At equilibrium, the sum of the moments about the center of the wheel (point O) must equal zero.

$$\sum M_O = 0$$

$$= rF_{\text{spring}} - WL\sin\theta$$

$$rF_{\text{spring}} = WL\sin\theta$$

$$F_{\text{spring}} = kr\theta$$

$$r^2k\theta = WL\sin\theta$$

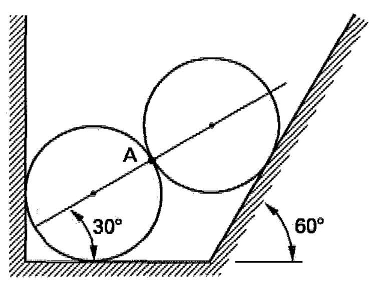
$$\theta = \frac{WL\sin\theta}{kr^2}$$

Successive iterations are necessary to solve for θ.

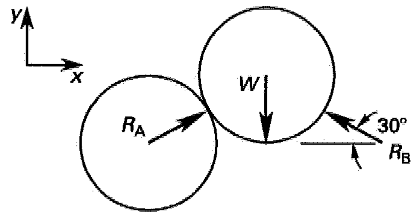
The answer is (B).

STATICS–10

Two identical spheres weighing 100 N each are placed as shown. The line connecting the two centers of the spheres makes an angle of 30° to the horizontal surface. All walls are smooth and frictionless. What is most nearly the reaction force at A?



(A) 33 N (B) 67 N (C) 75 N (D) 100 N



$$W = 100\ \text{N}$$

$$\sum F_x = 0$$

$$R_A \cos 30° - R_B \cos 30° = 0$$

$$R_A = R_B$$

$$\sum F_y = 0$$

$$-W + R_A \sin 30° + R_B \sin 30° = 0$$

$$R_A + R_B = \frac{W}{\sin 30°}$$

$$2R_A = \frac{W}{\sin 30°}$$

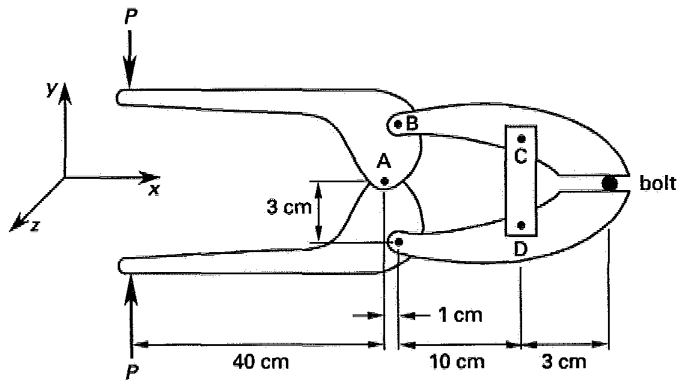
$$R_A = W$$

$$= 100\ \text{N}$$

The answer is (D).

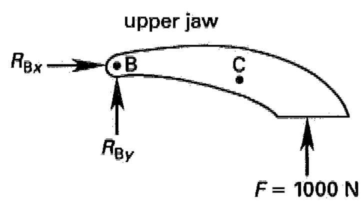
STATICS–11

What is most nearly the force, P, that must be exerted on the handles of the bolt cutter shown if the force on the bolt is 1000 N?



(A) 7.5 N (B) 30 N (C) 53 N (D) 260 N

First, consider the upper jaw.



$$\sum F_x = 0$$

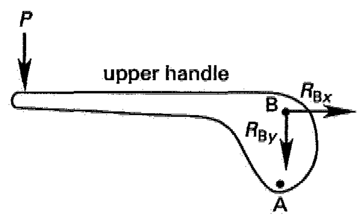
$$R_{Bx} = 0$$

$$\sum M_C = 0$$

$$-(R_{By})(10 \text{ cm}) + F(3 \text{ cm}) = 0$$

$$\begin{aligned} R_{By} &= \left(\frac{3 \text{ cm}}{10 \text{ cm}}\right) F \\ &= \left(\frac{3 \text{ cm}}{10 \text{ cm}}\right)(1000 \text{ N}) \\ &= 300 \text{ N} \end{aligned}$$

Now, consider the upper handle.



$$\sum M_A = 0$$

$$-R_{By}(1\text{ cm}) + P(40\text{ cm}) = 0$$

$$P = \left(\frac{1\text{ cm}}{40\text{ cm}}\right) R_{B_y}$$

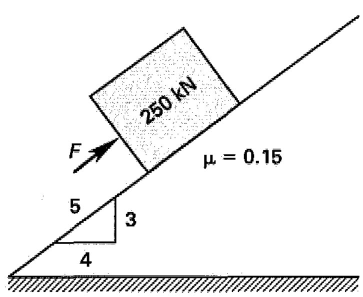
$$= \left(\frac{1\text{ cm}}{40\text{ cm}}\right)(300\text{ N})$$

$$= 7.5\text{ N}$$

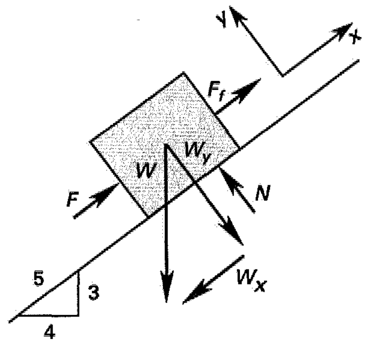
The answer is (A).

STATICS–12

Determine the force, F, required to keep the package from sliding down the plane shown.



(A) 15 kN (B) 35 kN (C) 65 kN (D) 120 kN

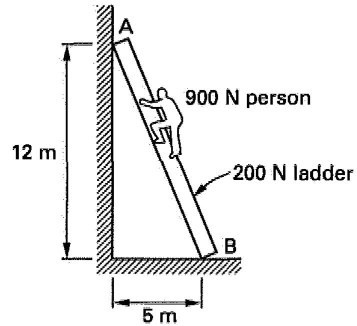


$$\sum F_y = 0$$
$$W_y - N = 0$$
$$W_y = \tfrac{4}{5}W$$
$$N = \tfrac{4}{5}W$$
$$= 200 \text{ kN}$$
$$F_f = \mu N$$
$$= (0.15)(200 \text{ kN})$$
$$= 30 \text{ kN}$$
$$\sum F_x = 0$$
$$F - W_x + F_f = 0$$
$$F = W_x - F_f$$
$$W_x = \tfrac{3}{5}W$$
$$= 150 \text{ kN}$$
$$F = 150 \text{ kN} - 30 \text{ kN}$$
$$= 120 \text{ kN}$$

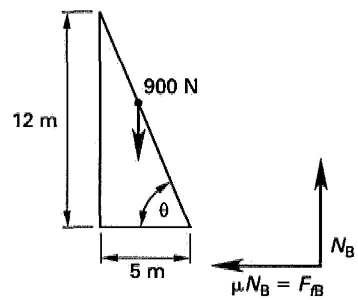
The answer is (D).

STATICS–13

Determine the minimum coefficient of friction at point B required for a person to use the ladder shown. Assume there is no friction at point A.



(A) 0.18 (B) 0.28 (C) 0.42 (D) 0.56



The total reaction force at B ($N_B + F_{fB}$) must point along the ladder. Therefore,

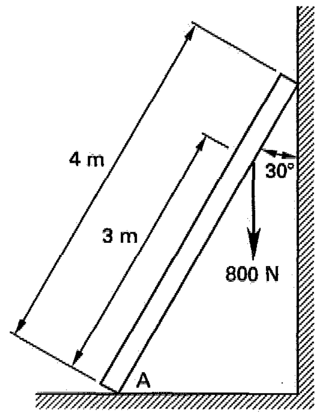
$$\frac{F_{fB}}{N_B} = \frac{\mu N_B}{N_B} = \cot\theta$$

$$\mu = \cot\theta = \frac{5 \text{ m}}{12 \text{ m}} = 0.42$$

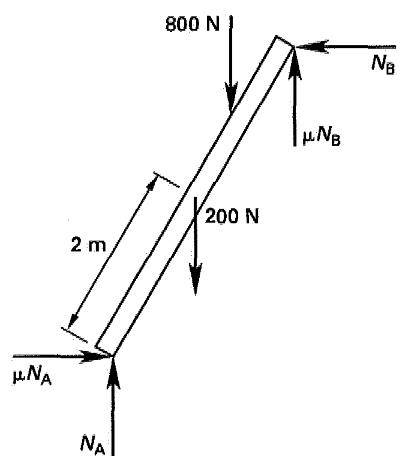
The answer is (C).

STATICS–14

A 4 m ladder weighing 200 N is placed as shown. When an 800 N person climbs 3 m above the lower end, the ladder is just about to slip. Determine the friction coefficient between the ladder and the floor. The coefficient of friction between the ladder and the wall is 0.20.



(A) 0.19 (B) 0.29 (C) 0.39 (D) 0.49



$$\sum M_A = 0$$

$$0 = (-200\ \text{N})(2\ \text{m})\sin 30° - (800\ \text{N})(3\ \text{m})\sin 30° + N_B(4\ \text{m})\cos 30° + 0.2N_B(4\ \text{m})\sin 30°$$

$$N_B = 362\ \text{N}$$

$$\sum F_y = 0$$

$$0 = N_A + \mu_{\text{wall}} N_B - 800\ \text{N} - 200\ \text{N}$$

$$= N_A + (0.20)(362\ \text{N}) - 800\ \text{N} - 200\ \text{N}$$

$$N_A = 928\ \text{N}$$

$$\sum F_x = 0$$

$$0 = \mu_{\text{floor}} N_A - N_B$$

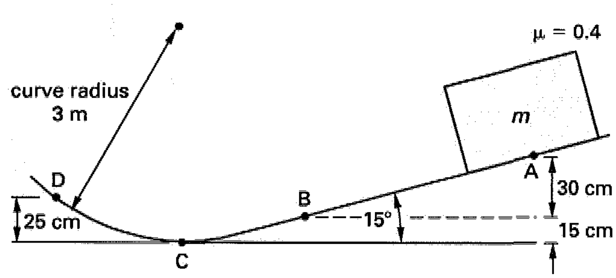
$$\mu_{\text{floor}} N_A = N_B$$

$$\mu_{\text{floor}} = \frac{N_B}{N_A} = \frac{362\ \text{N}}{928\ \text{N}} = 0.39$$

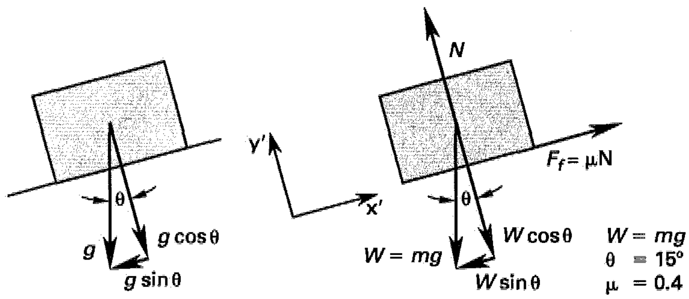
The answer is (C).

STATICS–15

A 2 kg block is released at point A on an inclined plane that is tangent to a circular arc. The plane is tilted 15° from horizontal, and the coefficient of friction is 0.4. Which point is the equilibrium position of the block?



(A) A (B) B (C) C (D) D



$$\sum F_{x'} = -mg\sin\theta + F_f$$
$$= -mg\sin\theta + \mu mg\cos\theta$$

For the block to slide downhill, $\sum F_x < 0$.

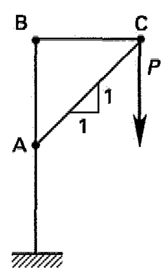
$$-mg\sin\theta + \mu mg\cos\theta < 0$$
$$\mu mg\cos\theta < mg\sin\theta$$
$$\mu < \tan\theta$$
$$< \tan 15°$$
$$< 0.27$$

Thus, for the block to move, $\mu < 0.27$. However, $\mu = 0.4$. Therefore, the block never moves. It stays at point A.

The answer is (A).

STATICS–16

Determine the force in member AC in terms of force P.



(A) P (B) $\frac{4}{3}P$ (C) $\sqrt{2}P$ (D) $\frac{\sqrt{3}}{2}P$

$$\text{AC} = \text{force in member AC}$$

$$\sum F_y = 0$$

$$0 = (\text{AC})\cos 45^\circ - P$$

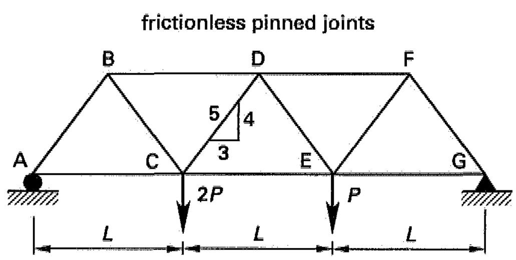
$$\text{AC} = \frac{P}{\cos 45^\circ}$$

$$= \sqrt{2}P$$

The answer is (C).

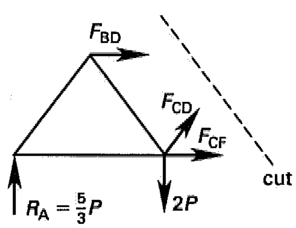
STATICS–17

Determine the force in member CD.



(A) $\frac{1}{12}P$ (B) $\frac{1}{3}P$ (C) $\frac{5}{12}P$ (D) P

Use the method of sections.



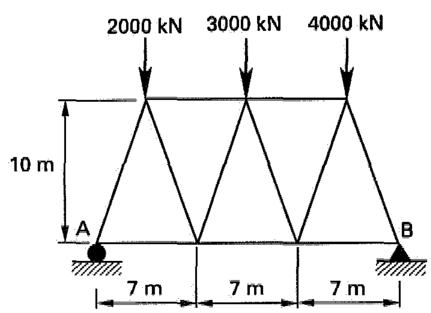
Only CD can support a vertical force.

$$\sum F_y = 0$$
$$0 = R_A - 2P + CD_y$$
$$CD_y = \frac{P}{3}$$
$$CD = \tfrac{5}{4}CD_y$$
$$= \left(\frac{5}{4}\right)\left(\frac{P}{3}\right)$$
$$= \frac{5P}{12}$$

The answer is (C).

STATICS–18

A truss is subjected to three loads. The truss is supported by a roller at A and by a pin joint at B. What is most nearly the reaction force at A?



(A) 3800 kN (B) 4400 kN (C) 4900 kN (D) 5000 kN

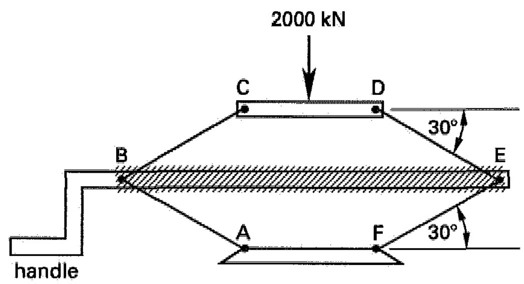
The rolling support at A can only support a vertical reaction force. R_A is the reaction force at A.

$$\sum M_B = 0$$
$$0 = -R_A(21 \text{ m}) + (2000 \text{ kN})(17.5 \text{ m}) + (3000 \text{ kN})(10.5 \text{ m}) + (4000 \text{ kN})(3.5 \text{ m})$$
$$R_A = 3833 \text{ kN} \quad (3800 \text{ kN})$$

The answer is (A).

STATICS–19

A scissor jack is used to raise a car. If the jack supports a weight of 2000 kN, determine the force in member BE.



(A) $\sqrt{3} \times 100$ kN (B) $\sqrt{3} \times 500$ kN (C) $\sqrt{3} \times 1000$ kN (D) $\sqrt{3} \times 2000$ kN

CB and DE equally share the 2000 kN load. Therefore, $BC_y = DE_y = 2000 \text{ kN}/2 = 1000$ kN, and $DE = DE_y/\sin 30° = 1000 \text{ kN}/0.5 = 2000$ kN. Use the method of joints at E.

$$\sum F_y = 0$$

$$0 = -FE_y + DE_y$$

$$FE_y = 1000 \text{ kN}$$

$$FE = \frac{FE_y}{\sin 30°} = \frac{1000 \text{ kN}}{0.5} = 2000 \text{ kN}$$

$$\sum F_x = 0$$

Therefore,

$$BE = FE_x + DE_x$$

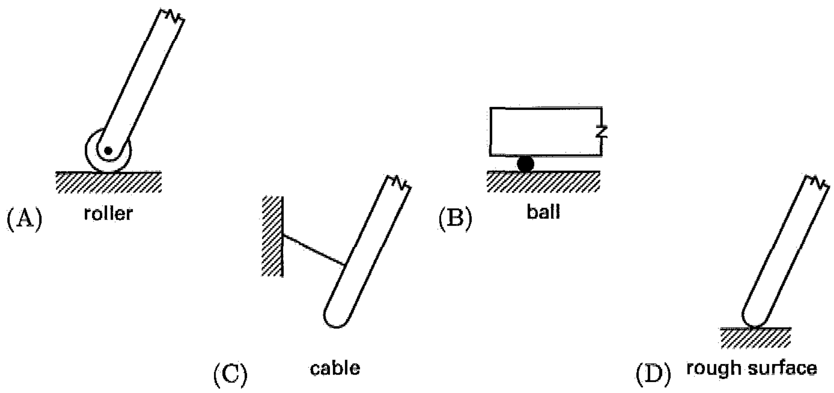
$$FE \cos 30° + DE \cos 30° = (2)(2000 \text{ kN})\left(\frac{\sqrt{3}}{2}\right)$$

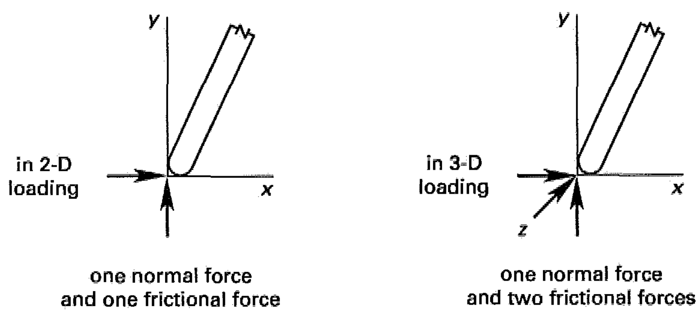
$$= \sqrt{3} \times 2000 \text{ kN}$$

The answer is (D).

STATICS–20

When loaded, which support has a reaction involving more than a single force?



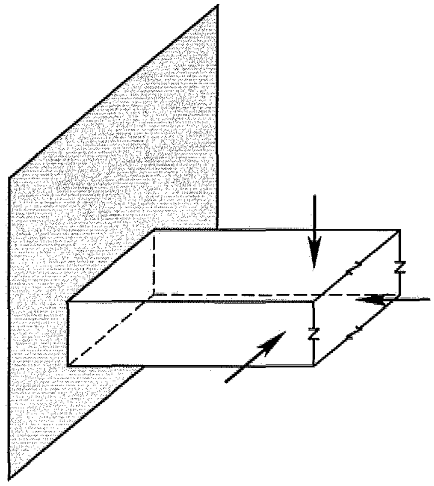


Options (A) and (B) have a normal force. Option (C) has a single force along the cable. Only option (D) can support two reaction forces.

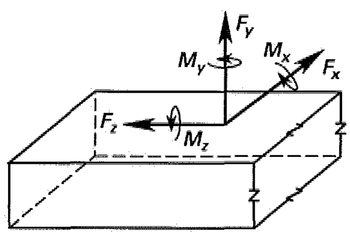
The answer is (D).

STATICS–21

A beam, securely fastened to a wall, is subjected to three-dimensional loading. How many components of reaction are possible?



(A) two (B) three (C) four (D) six

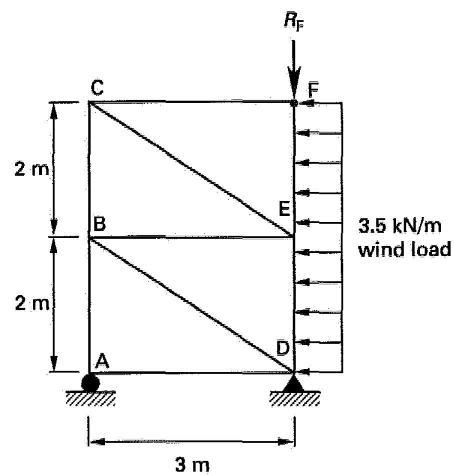


There are six components of reaction possible: three force components and three moment components.

The answer is (D).

STATICS–22

What is most nearly the vertical force at point F necessary to resist the wind load of 3.5 kN/m on DEF?



(A) 2.2 kN (B) 4.3 kN (C) 9.3 kN (D) 10 kN

$$\sum M_A = 0$$

$$0 = \left(3.5\ \frac{\text{kN}}{\text{m}}\right)(4\ \text{m})(2\ \text{m}) - (3\ \text{m})R_F$$

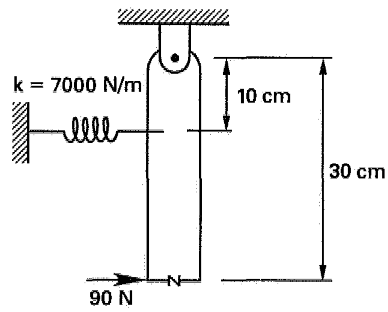
Rearranging to solve for R_F,

$$R_F = \frac{28\ \text{kN·m}}{3\ \text{m}}$$
$$= 9.33\ \text{kN} \quad (9.3\ \text{kN})$$

The answer is (C).

STATICS–23

Compute the equilibrium displacement of the spring shown.



(A) 12 mm (B) 32 mm (C) 38 mm (D) 75 mm

Sum the moments around the hinge.

$$\sum M_{\text{hinge}} = 0 = (90\ \text{N})(30\ \text{cm}) - F_{\text{spring}}(10\ \text{cm})$$

$$F_{\text{spring}} = 270\ \text{N} = k\delta_{\text{spring}}$$

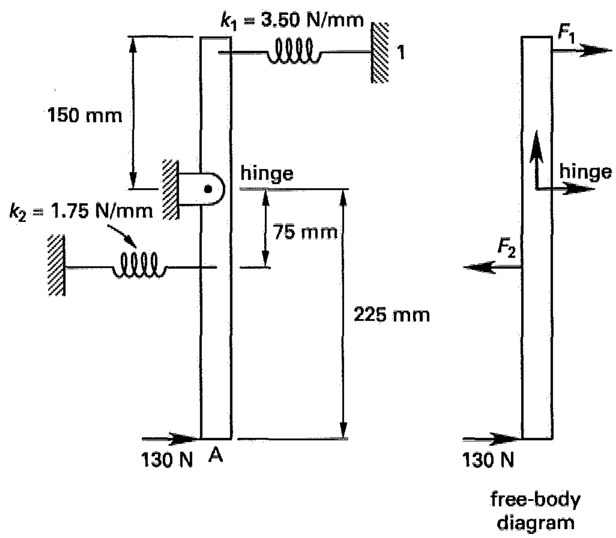
$$\delta_{\text{spring}} = \frac{F_{\text{spring}}}{k} = \frac{270\ \text{N}}{7000\ \frac{\text{N}}{\text{m}}}$$

$$= 0.038\ \text{m} \quad (38\ \text{mm})$$

The answer is (C).

STATICS–24

The system shown is in equilibrium prior to the application of the 130 N force at A. After equilibrium is reestablished, what is most nearly the displacement at A?



(A) 25 mm (B) 30 mm (C) 37 mm (D) 74 mm

Sum the moments around the hinge and use $F = -k\delta$ to find the spring forces.

$$\sum M_{\text{hinge}} = 0$$

$$0 = -k_1\delta_1(150 \text{ mm}) + (130 \text{ N})(225 \text{ mm}) - k_2\delta_2(75 \text{ mm})$$

The ratio of the deflection of the bar at a point to that point's distance from the hinge is equal to the angular displacement of the bar. Since the bar does not bend, this ratio is the same for any point on the bar. Therefore,

$$\frac{\delta_1}{150 \text{ mm}} = \frac{\delta_2}{75 \text{ mm}}$$

$$\delta_1 = 2\delta_2$$

Substitute for δ_1 in the equation for the moment.

$$0 = -\left(3.5\ \frac{\text{N}}{\text{mm}}\right)2\delta_2(150 \text{ mm}) + (130 \text{ N})(225 \text{ mm}) - \left(1.75\ \frac{\text{N}}{\text{mm}}\right)\delta_2(75 \text{ mm})$$

Rearranging to solve for δ_2,

$$\delta_2 = \frac{(130\ \text{N})(225\ \text{mm})}{1181.3\ \text{N}} = 24.8\ \text{mm}$$

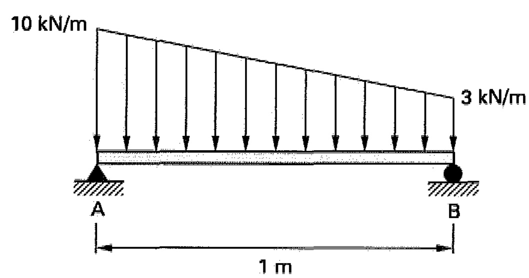
$$\frac{\delta_A}{225\ \text{mm}} = \frac{\delta_2}{75\ \text{mm}}$$

$$\delta_A = 3\delta_2 = 74.4\ \text{mm} \quad (74\ \text{mm})$$

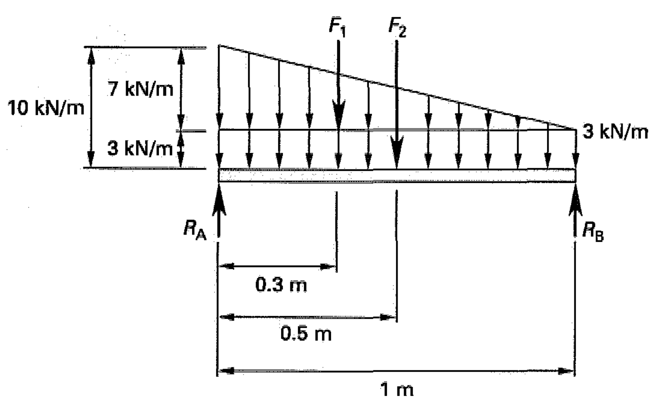
The answer is (D).

STATICS–25

What is most nearly the reaction force at support B on the simply supported beam with a linearly varying load?



(A) 1.5 kN (B) 2.3 kN (C) 2.6 kN (D) 3.5 kN



$$F_1 = \tfrac{1}{2}Lh = \left(\frac{1}{2}\right)(1\ \text{m})\left(7\ \frac{\text{kN}}{\text{m}}\right)$$
$$= 3.5\ \text{kN}$$
$$F_2 = Lh = (1\ \text{m})\left(3\ \frac{\text{kN}}{\text{m}}\right)$$
$$= 3\ \text{kN}$$

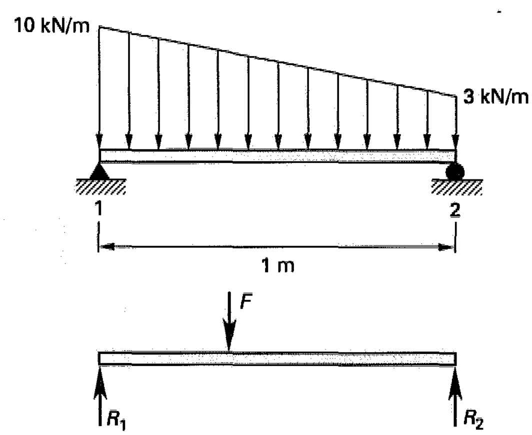
Sum the moments around support A.

$$\sum M_A = 0 = R_B(1\ \text{m}) - F_1(0.3\ \text{m}) - F_2(0.5\ \text{m})$$
$$= R_B(1\ \text{m}) - (3.5\ \text{kN})(0.3\ \text{m}) - (3\ \text{kN})(0.5\ \text{m})$$
$$R_B = 2.55\ \text{kN} \quad (2.6\ \text{kN})$$

The answer is (C).

STATICS–26

For the simply supported beam with the linearly varying load shown, what is the sum of the reactions at the supports?



free-body diagram

(A) 3.5 kN (B) 6.5 kN (C) 9.2 kN (D) 13 kN

$$\sum F_y = 0$$

$$R_1 + R_2 - F = 0$$

$$R_1 + R_2 = F$$

The area of the trapezoidal load distribution, F, is

$$F = \tfrac{1}{2}Lh = \left(\frac{10\ \frac{\text{kN}}{\text{m}} + 3\ \frac{\text{kN}}{\text{m}}}{2}\right)(1\ \text{m})$$

$$= 6.5\ \text{kN}$$

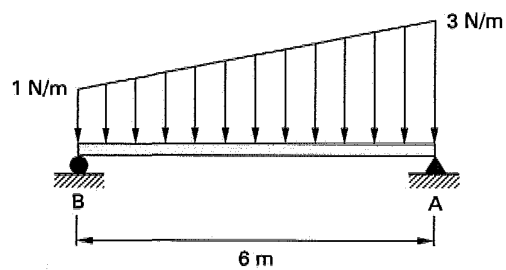
Therefore,

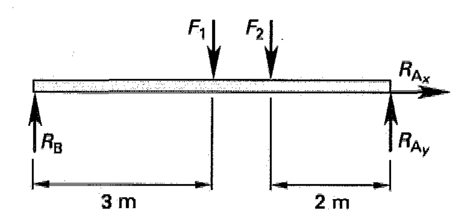
$$R_1 + R_2 = 6.5\ \text{kN}$$

The answer is (B).

STATICS–27

A beam is subjected to a distributed load as shown. Determine the reactions at the right support, A.





free-body diagram

(A) 2.2 kN (B) 4.3 kN (C) 5.5 kN (D) 7.0 kN

$$\sum F_x = 0$$

$$R_{Ax} = 0$$

$$F_1 = Lh = \left(1\ \frac{\text{N}}{\text{m}}\right)(6\ \text{m}) = 6\ \text{N}$$

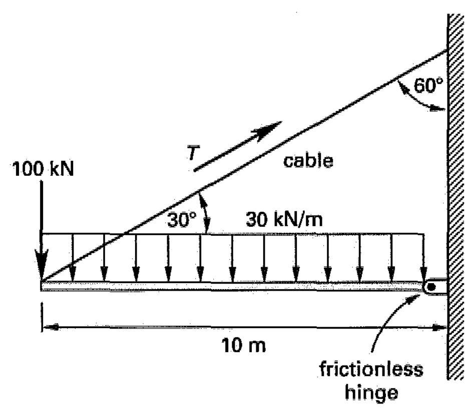
$$F_2 = \tfrac{1}{2}Lh = \left(\frac{1}{2}\right)\left(2\ \frac{\text{N}}{\text{m}}\right)(6\ \text{m}) = 6\ \text{N}$$

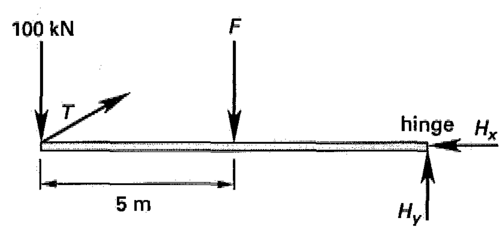
Since $R_{Ax} = 0$, $R_{Ay} = R_A$. Thus, $R_A = 7.0$ kN.

The answer is (D).

STATICS–28

A beam is hinged at a wall and loaded as shown. What is the tension in the cable?





free-body diagram

(A) 200 kN (B) 250 kN (C) 430 kN (D) 500 kN

$$\sum m_{\text{hinge}} = 0$$

$$F(5\text{ m}) + (100\text{ kN})(10\text{ m}) - T\sin 30°(10\text{ m}) = 0$$

$$F = Lh = (10\text{ m})\left(30\ \frac{\text{kN}}{\text{m}}\right) = 300\text{ kN}$$

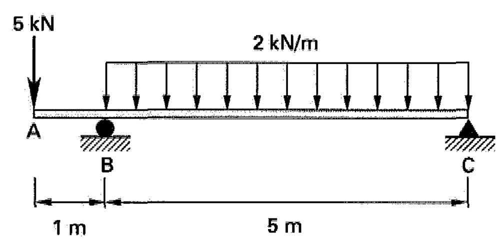
Substituting for F and rearranging to solve for T,

$$T = \frac{(300\text{ kN})(5\text{ m}) + (100\text{ kN})(10\text{ m})}{(10\text{ m})(\sin 30°)} = 500.0\text{ kN}$$

The answer is (D).

STATICS–29

Determine the position of maximum moment in the beam ABC.



(A) at point A (B) at point B (C) at point C (D) 2 m left of point C

$$\sum M_B = 0$$

$$0 = (5\text{ kN})(1\text{ m}) - \int_0^5 2x\,dx + (R_{C_y})(5\text{ m})$$

$$= 5\text{ kN·m} - x^2\Big|_0^5 + R_{C_y}(5\text{ m})$$

$$= 5\text{ kN·m} - (25 - 0) + R_{C_y}(5\text{ m})$$

$$R_{C_y} = 4\text{ kN}$$

$$\sum F_y = 0 = -5\text{ kN} - \int_0^5 2dx + 4\text{ kN} + R_B$$
$$= -5\text{ kN} - 2x\Big|_0^5 + 4\text{ kN} + R_B$$
$$= -5\text{ kN} - (10-0) + 4\text{ kN} + R_B$$
$$= -5\text{ kN} - 10\text{ kN} + 4\text{ kN} + R_B$$
$$R_B = 11\text{ kN}$$

At point A,

$$M_A = 0$$

At point B,

$$M_B = (5\text{ kN})(1\text{ m}) = 5\text{ kN·m}$$

From C to B,

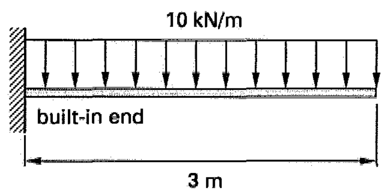
$$M_x = R_{Cy}x - \int_0^x 2xdx$$
$$= \left(4x - \frac{2x^2}{2}\right)$$
$$= (4x - x^2)$$
$$\frac{dM_x}{dx} = 4 - 2x$$
$$= 0 \quad [\text{where } M_x \text{ is a max}]$$
$$0 = (4\text{ kN}) - \left(2\,\frac{\text{kN}}{\text{m}}\right)x$$
$$x = \frac{4\text{ kN}}{2\,\frac{\text{kN}}{\text{m}}}$$
$$= 2\text{ m}$$
$$M_{x,\text{max}} = (4\text{ kN})(2\text{ m}) - 4\text{ kN·m}$$
$$= 4\text{ kN·m} < M_B$$

Thus, the maximum moment is 5 kN·m, and it occurs at point B.

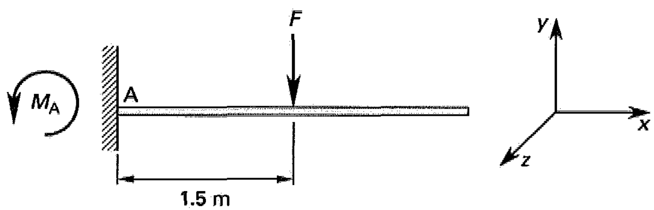
The answer is (B).

STATICS–30

For the cantilever beam with the distributed load shown, what is the moment at the built-in end?



(A) 5 kN·m (B) 10 kN·m (C) 20 kN·m (D) 45 kN·m

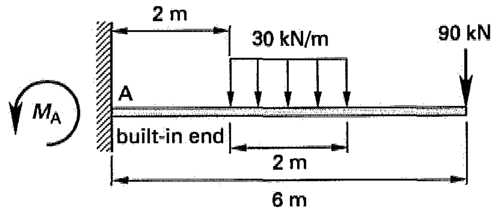


$$F = \left(10\ \frac{\text{kN}}{\text{m}}\right)(3\ \text{m})$$
$$= 30\ \text{kN}$$
$$\sum M_A = 0 = M_A - F(1.5\ \text{m})$$
$$M_A = (30\ \text{kN})(1.5\ \text{m})$$
$$= 45\ \text{kN·m}$$

The answer is (D).

STATICS–31

For the cantilever beam shown, what is the moment acting at the built-in end?



(A) 270 kN·m (B) 310 kN·m (C) 540 kN·m (D) 720 kN·m

Sum the moments around the support, A.

$$\sum M_A = 0 = M_A - (90 \text{ kN})(6 \text{ m}) - \left(30 \ \frac{\text{kN}}{\text{m}}\right)(2 \text{ m})\left(\frac{6 \text{ m}}{2}\right)$$

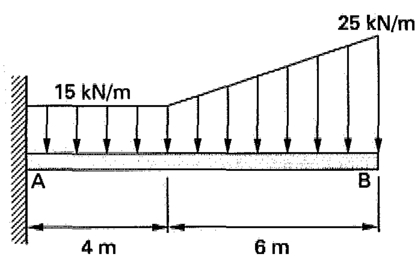
Rearranging to solve for M_A,

$$M_A = 720 \text{ kN·m}$$

The answer is (D).

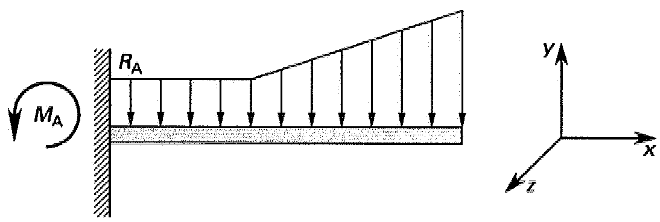
STATICS–32

Determine the resultant moment at the built-in end, A, for the cantilever beam shown.



(A) 660 kN·m (B) 990 kN·m (C) 1100 kN·m (D) 1200 kN·m

Divide the beam into two sections: from 0 m to 4 m where $F_1 = 15$ kN·m, and from 4 m to 10 m where $F_2 = 5(x+5)/3$ kN/m. Sum the moments around point A.



$$\sum M_A = 0$$

$$0 = M_A - \int_0^4 F_1(x)x\,dx - \int_4^{10} F_2(x)x\,dx$$

$$M_A = \int_0^4 15x\,dx + \int_4^{10} \frac{5(x+5)}{3}x\,dx$$

$$= \left.\frac{15x^2}{2}\right|_0^4 + \left.\frac{5x^3}{9} + \frac{25x^2}{6}\right|_4^{10}$$

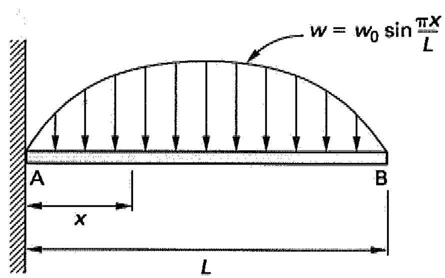
$$= (15)(8) + \frac{(5)\left(10^3 - (4)^3\right)}{9} + \frac{(25)\left(10^2 - (4)^2\right)}{6}$$

$$= 990 \text{ kN·m}$$

The answer is (B).

STATICS–33

Determine the moment at the built-in end, A, for the beam shown.



(A) $\frac{w_0 L^2}{\pi}$ (B) $w_0 L$ (C) $\frac{2w_0 L^2}{\pi}$ (D) $\frac{w_0 L^2}{2\pi}$

$$\sum M_A = 0$$

$$0 = M_A - \int_0^L \left(w_0 \sin\frac{\pi x}{L}\right) x\,dx$$

$$M_A = \int_0^L \left(w_0 \sin\frac{\pi x}{L}\right) x\,dx$$

Integrate by parts.

$$u = x$$

$$dv = \left(w_0 \sin\frac{\pi x}{L}\right) dx$$

$$du = dx$$

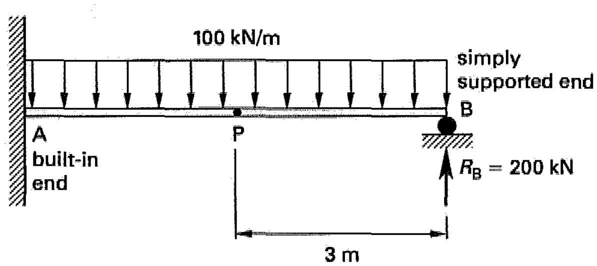
$$v = -\frac{w_0 L}{\pi}\cos\frac{\pi x}{L}$$

$$\begin{aligned} M_A &= -\frac{x w_0 L}{\pi}\cos\frac{\pi x}{L}\bigg|_0^L + \frac{w_0 L}{\pi}\int_0^L \left(\cos\frac{\pi x}{L}\right) dx \\ &= -\left(\frac{L^2 w_0}{\pi}\cos\pi - 0\right) + \left(\left(\frac{w_0 L^2}{\pi^2}\right)\sin\frac{\pi x}{L}\bigg|_0^L\right) \\ &= -\left(\frac{L^2 w_0}{\pi}\right)(-1 - 0) + \left(\frac{w_0 L^2}{\pi^2}\right)(0 - 0) \\ &= \frac{L^2 w_0}{\pi} \end{aligned}$$

The answer is (A).

STATICS–34

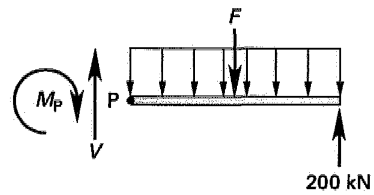
The beam shown is statically indeterminate, but the reaction force at B is known to be 200 kN. What is the bending moment at point P?



(A) 90 kN·m (B) 150 kN·m (C) 240 kN·m (D) 350 kN·m

Measure x from point B.

The moment at point P is



$$\sum M_P = 0 = -M_P - F(1.5 \text{ m}) + (200 \text{ kN})(3 \text{ m})$$

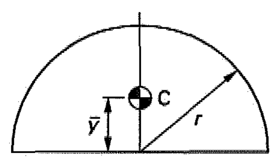
Rearranging to solve for M_P,

$$\begin{aligned} M_P &= (200 \text{ kN})(3 \text{ m}) - \left(100 \ \frac{\text{kN}}{\text{m}}\right)(3 \text{ m})(1.5 \text{ m}) \\ &= 150 \text{ kN·m} \end{aligned}$$

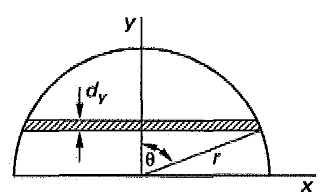
The answer is (B).

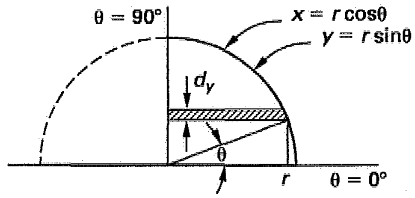
STATICS–35

Determine the height of the centroid, $\bar{y}$, of the semicircle with radius r shown.



(A) $\frac{2r}{3}$ (B) $\frac{2\pi r}{5}$ (C) $\frac{4r}{3\pi}$ (D) $\frac{3r}{4}$





$$A = \tfrac{1}{2}\pi r^2 \quad \text{[for the semicircle]}$$

$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$dy = d(r\sin\theta) = r\cos\theta d\theta$$

$$dA = xdy = r\cos\theta r\cos\theta d\theta$$

Since the area is symmetrical, area for $0 \leq \theta \leq 90°$ is half the total area.

$$\begin{aligned}\int ydA &= 2\int_0^{\pi/2}(r\sin\theta)(r\cos\theta)(r\cos\theta)d\theta \\ &= 2\int_0^{\pi/2} r^3\sin\theta\cos^2\theta d\theta \\ &= \frac{-2r^3\cos^3\theta}{3}\bigg|_0^{\pi/2} \\ &= \frac{2r^3}{3}\end{aligned}$$

By definition,

$$\bar{y}A = \int_A ydA$$

$$\bar{y} = \frac{\int_A^y dA}{A}$$

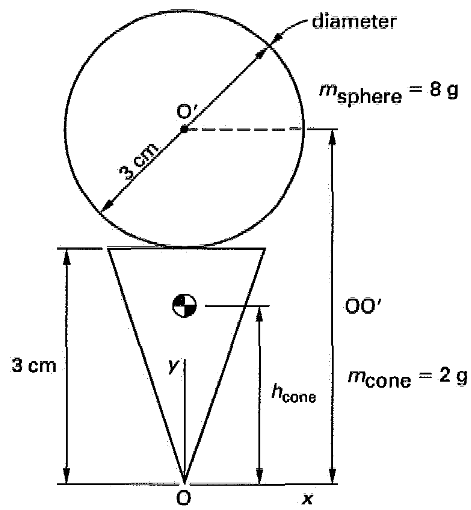
$$\bar{y}A = \frac{2r^3}{3}$$

$$\bar{y} = \frac{\frac{2r^3}{3}}{\frac{\pi r^2}{2}} = \frac{4r}{3\pi}$$

The answer is (C).

STATICS–36

What is the height of the center of mass of the cone-sphere system shown?



(A) 2 cm (B) 3 cm (C) 4 cm (D) 5 cm

By symmetry, the center of mass of the system is on the y-axis. Find the height of the center of mass for each part of the system.

First, find the height of the sphere's center of mass, OO′.

$$\begin{aligned} \text{OO}' &= \text{height of cone} + r_{\text{sphere}} \\ &= 3 \text{ cm} + \frac{3 \text{ cm}}{2} \\ &= 4.5 \text{ cm} \end{aligned}$$

Next, find the height of the cone's center of mass, h_c.

$$\begin{aligned} h_c &= (\text{height of cone})\left(\frac{2}{3}\right) \\ &= (3 \text{ cm})\left(\frac{2}{3}\right) \\ &= 2 \text{ cm} \end{aligned}$$

Finally, find the height of the cone-sphere system's center of mass.

$$\begin{aligned} h &= \frac{m_{\text{sphere}}\text{OO}' + m_{\text{cone}}h_c}{m_{\text{sphere}} + m_{\text{cone}}} \\ &= \frac{(8\text{ g})(4.5\text{ cm}) + (2\text{ g})(2\text{ cm})}{8\text{ g} + 2\text{ g}} \\ &= 4\text{ cm} \end{aligned}$$

The answer is (C).

STATICS–37

Which statement about area moments of inertia is FALSE?

(A) $I = \int d^2 dA$

(B) The parallel axis theorem is used to calculate moments of inertia about a parallel displaced axis.

(C) The moment of inertia of a large area is equal to the summation of the inertia of the smaller areas within the large area.

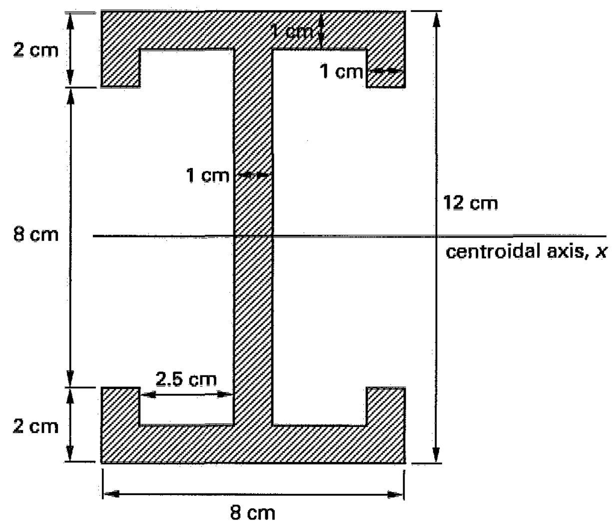
(D) The areas closest to the axis of interest contribute most to the moment of inertia.

Area moment of inertia is defined as $I = \int d^2 dA$, where d is the distance from the axis to the area element. Thus, the areas farthest from the axis have the largest contributions.

The answer is (D).

STATICS–38

Determine the moment of inertia around the centroidal axis of the following beam.

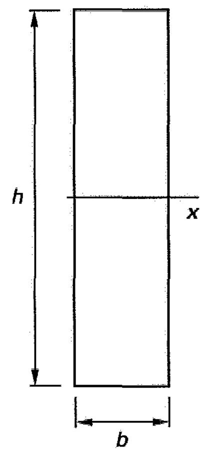


(A) 420 cm^4 (B) 650 cm^4 (C) 730 cm^4 (D) 950 cm^4

The moment of inertia of the beam is equivalent to the moment of inertia of a solid beam with the same dimensions (a height of 12 cm and a width of 8 cm) minus the moments of inertia of the missing sections.

The moment of inertia about the centroidal axis of rectangular sections is

$$I_x = \frac{bh^3}{12}$$



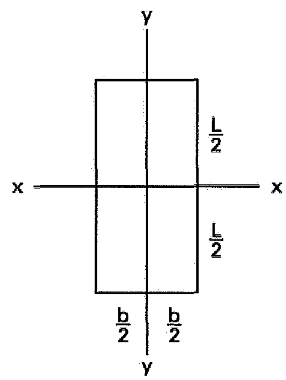
This equation is applied as follows for the solid section (8 cm × 12 cm), the two removed sections (2.5 cm × 10 cm each), and two other removed sections (1 cm × 8 cm each).

$$I_x = \left(\frac{1}{12}\right)(8\text{ cm})(12\text{ cm})^3 - (2)\left(\left(\frac{1}{12}\right)(2.5\text{ cm})(10\text{ cm})^3\right) - (2)\left(\left(\frac{1}{12}\right)(1\text{ cm})(8\text{ cm})^3\right) = 650\text{ cm}^4$$

The answer is (B).

STATICS–39

I_{xx} is the moment of inertia of the plane area about its centroidal x axis. How can I_{xx} be expressed?



(A) $\frac{bL^3}{96}$ (B) $\frac{bL^3}{16}$ (C) $\frac{bL^3}{12}$ (D) $\frac{bL^3}{8}$

$$I_{xx} = \int y^2 dA$$
$$= \int_{-\frac{L}{2}}^{\frac{L}{2}} y^2 b\, dy$$
$$= b \cdot \frac{y^3}{3}\Bigg|_{-\frac{L}{2}}^{\frac{L}{2}}$$
$$= b\left(\frac{\left(\frac{L}{2}\right)^3}{3} - \frac{\left(\frac{-L}{2}\right)}{3}\right)$$
$$= \frac{b}{3}\left(\frac{2L^3}{8}\right)$$
$$= \frac{bL^3}{12}$$

The answer is (C).