UNIVERSITY OF
SOUTHERN CALIFORNIA

Department of Civil Engineering

EERA

A Computer Program for
Equivalent-linear **E**arthquake site **R**esponse **A**nalyses
of Layered Soil Deposits

by

J. P. BARDET, K. ICHII, and C. H. LIN

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1. INTRODUCTION

During past earthquakes, the ground motions on soft soil sites were found to be generally larger than those of nearby rock outcrops, depending on local soil conditions (e.g., Seed and Idriss, 1968). These amplifications of soil site responses were simulated using several computer programs that assume simplified soil deposit conditions such as horizontal soil layers of infinite extent. One of the first computer programs developed for this purpose was SHAKE (Schnabel et al., 1972). More than 25 years after its release, SHAKE is still commonly used and referenced computer programs in geotechnical earthquake engineering. SHAKE computes the response in a horizontally layered soil-rock system subjected to transient and vertical travelling shear waves. SHAKE is based on the wave propagation solutions of Kanai (1951), Roesset and Whitman (1969), and Tsai and Housner (1970). SHAKE assumes that the cyclic soil behavior can be simulated using an equivalent linear model, which is extensively described in the geotechnical earthquake engineering literature (e.g., Idriss and Seed, 1968; Seed and Idriss, 1970; and Kramer, 1996). SHAKE was modified many times (e.g., frequency-dependent equivalent strain; Sugito, 1995). SHAKE91 is one of the most recent versions of SHAKE (Idriss and Sun, 1992).

In 1998, the computer program EERA was developed in FORTRAN 90 starting from the same basic concepts as SHAKE. EERA stands for Equivalent-linear Earthquake Response Analysis. EERA is a modern implementation of the well-known concepts of equivalent linear earthquake site response analysis. EERA's implementation takes full advantages of the dynamic array dimensioning and matrix operations in FORTRAN 90. EERA's input and output are fully integrated with the spreadsheet program Excel.

Following the introduction, the second section of this report reviews the theory of equivalent linear soil model, and one-dimensional ground response analysis. The two appendices contain a sample problem and comparison between EERA and SKAKE91 results.

2. EQUIVALENT LINEAR MODEL FOR SOIL RESPONSE

2.1 One-dimensional Stress-Strain Relationship

The equivalent linear model represents the soil stress-strain response based on a Kelvin-Voigt model as illustrated in Fig. 1. The shear stress τ depends on the shear strain γ and its rate $\dot{\gamma}$ as follows:

$$\tau = G\gamma + \eta\dot{\gamma} \qquad (1)$$

where G is shear modulus and η the viscosity. In a one-dimensional shear beam column, the shear strain and its rate are defined from the horizontal displacement u(z,t) at depth z and time t as follows:

$$\gamma = \frac{\partial u(z,t)}{\partial z} \text{ and } \dot{\gamma} = \frac{\partial \gamma(z,t)}{\partial t} = \frac{\partial^2 u(z,t)}{\partial z \partial t} \qquad (2)$$

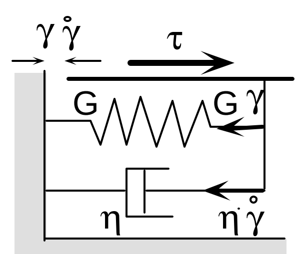


Figure 1. Schematic representation of stress-strain model used in equivalent-linear model.

In the case of harmonic motion, the displacement, strain and strain rate are:

$$u(z,t) = U(z)e^{i\omega t}\ ,\ \gamma(z,t) = \frac{dU}{dz}e^{i\omega t} = \Gamma(z)e^{i\omega t} \text{ and } \dot{\gamma}(z,t) = i\omega\gamma(z,t) \tag{3}$$

where $U(z)$ and $\Gamma(z)$ are the amplitude of displacement and shear strain, respectively. Using Eq. 3, the stress-strain relation (i.e., Eq. 1) becomes in the case of harmonic loadings:

$$\tau(z,t) = \Sigma(z)e^{i\omega t} = (G + i\omega\eta)\frac{dU}{dz}e^{i\omega t} = G^*\frac{dU}{dz}e^{i\omega t} = G^*\gamma(z,t) \tag{4}$$

where G^* is the complex shear modulus and $\Sigma(z)$ is the amplitude of shear stress. After introducing the critical damping ratio ξ so that $\xi = \omega\eta/2G$, the complex shear modulus G^* becomes:

$$G^* = G + i\omega\eta = G(1 + 2i\xi) \tag{5}$$

The energy dissipated W_d during a complete loading cycle is equal to the area generated by the stress-strain loop, i.e.:

$$W_d = \oint_{\tau_c} \tau\, d\gamma \tag{6}$$

In the case of strain-controlled harmonic loading of amplitude γ_c (i.e., $\gamma(t) = \gamma_c e^{i\omega t}$), Eq. 6 becomes:

$$W_d = \int_t^{t+2\pi/\omega} \mathrm{Re}[\tau(t)]\mathrm{Re}\left[\frac{d\gamma}{dt}\right]dt \tag{7}$$

where only the real parts of the stress and strain rate are considered (Meirovitch, 1967). Using Eq. 4, the real parts of stress and strain rate are:

$$\mathrm{Re}[\tau(t)] = \gamma_c(G\cos\omega t - \omega\eta\sin\omega t) \text{ and } \mathrm{Re}\left[\frac{d\gamma}{dt}\right] = -\gamma_c\omega\sin\omega t \tag{8}$$

Finally, Eq. 7 becomes:

$$W_d = \frac{1}{2}\omega\gamma_c^2\int_t^{t+2\pi/\omega}[-G\sin 2\omega t + \omega\eta(1 - \cos 2\omega t)]dt = \pi\omega\eta\gamma_c^2 \tag{9}$$

The maximum strain energy stored in the system is:

$$W_s = \frac{1}{2}\tau_c\gamma_c = \frac{1}{2}G\gamma_c^2 \tag{10}$$

The critical damping ratio ξ can be expressed in terms of W_d and W_s as follows:

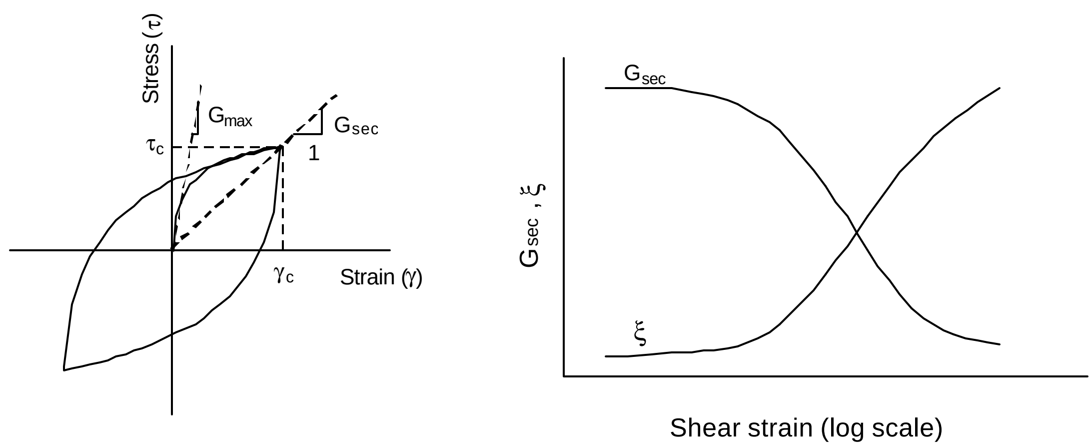
$$\xi = \frac{W_d}{4\pi W_s} \tag{11}$$

2.2 Equivalent Linear Approximation of Nonlinear Stress-strain Response

The *equivalent linear* approach consists of modifying the Kelvin-Voigt model to account for some types of soil nonlinearities. The nonlinear and hysteretic stress-strain behavior of soils is approximated during cyclic loadings as shown in Fig. 2. The equivalent linear shear modulus, *G*, is taken as the secant shear modulus G_s, which depends on the shear strain amplitude g. As shown in Fig. 2a, G_s at the ends of symmetric strain-controlled cycles is:

$$G_s = \frac{\boldsymbol{t}_c}{\boldsymbol{g}_c} \tag{12}$$

where t_c and g_c are the shear stress and strain amplitudes, respectively. The equivalent linear damping ratio, ξ, is the damping ratio that produces the same energy loss in a single cycle as the hysteresis stress-strain loop of the irreversible soil behavior. Examples of data for equivalent linear model can be found in Hardin and Drnevitch (1970), Kramer (1996), Seed and Idriss (1970), Seed et al. (1986), Sun et al. (1988), and Vucetic and Dobry (1991).



(a) (b)

Figure 2. Equivalent-linear model: (a) Hysteresis stress-strain curve; and (b) Variation of secant shear modulus and damping ratio with shear strain amplitude.

In site response analysis, the material behavior is generally specified as shown in Fig. 2b. The G_s-g curves cannot have arbitrary shapes as they derive from t-g stress-strain curves. For instance, the exclusion of strain-softening from the t-g curves imposes some restrictions on the corresponding G_s-g curves. Strain softening is a real physical phenomen which corresponds to a decrease in stress with strain. Its inclusion requires complicated numerical techniques that are beyond the scope of most enginering site responses. Without these special techniques, strain-softening has been shown to create ill-posed boundary value problems, and undesirable numerical effects such as numerical solutions that dependent strongly on spatial discretization (i.e., mesh geometry). The exclusion of strain-softening implies that:

$$\frac{d\boldsymbol{t}}{dg} = G_s(g) + \frac{dG_s}{dg} g \geq 0 \tag{13}$$

In the case of G_s-g curves specified with discrete points (G_i, g_i), Eq. 13 becomes:

$$\frac{\Delta G_s}{G_{\max}} \geq -\frac{G_s(g)}{G_{\max}} \frac{\Delta g}{g} \tag{14}$$

where DG_s is the decrease in G_s corresponding to the increase Dg in g, and G_{max} is the maximum value of G_s. Eq. 14 is equivalent to:

$$G_{i+1}/G_i \geq 2 - g_{i+1}/g_i \tag{15}$$

Figure 3 illustrates a particular case of strain-softening, which is clearly visible on a stress-strain response but difficult to detect from a G_s-γ curve.

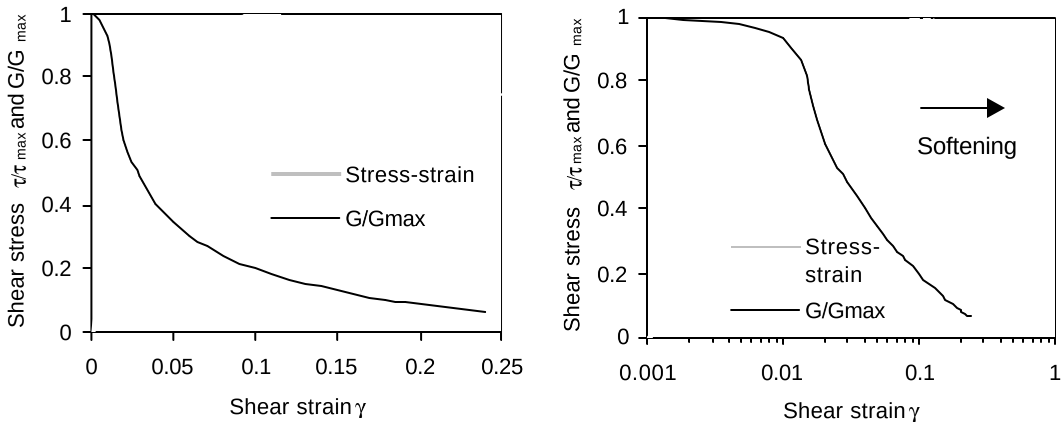


Figure 3. An example of strain softening in τ/τ_{max} and G/G_{max} curves plotted in linear and semi-logarithmic scales.

As shown in Fig. 2b, the equivalent linear model specifies the variation of shear modulus and damping ratio with shear strain amplitude. Additional assumptions are required to specify the effects of frequency on stress-strain relations. For this purpose, two basic models have been proposed.

2.2.1 Model 1

Model 1 is used in the original version of SHAKE (Schnabel et al., 1972). It assumes that ξ is constant and independent of ω, which implies that the complex modulus G^* is also independent of ω. The dissipated energy during a loading cycle is:

$$W_d = 4\pi W_s \xi = 2\pi \xi G \gamma_c^2 = \pi \eta \gamma_c^2 \omega \tag{16}$$

The dissipated energy increases linearly with ξ, and is independent of ω, which implies that the area of stress-strain loops is frequency independent. The amplitudes of complex and real shear modulus are related through:

$$\left|G^*\right| = G\sqrt{1 + 4\xi^2} \tag{17}$$

which implies that $|G^*|$ increases with ξ. Figure 4 shows the variation of $|G^*|/G$ with ξ. The amplitude of the complex shear modulus can vary as much as 12 % when ξ reaches 25%.

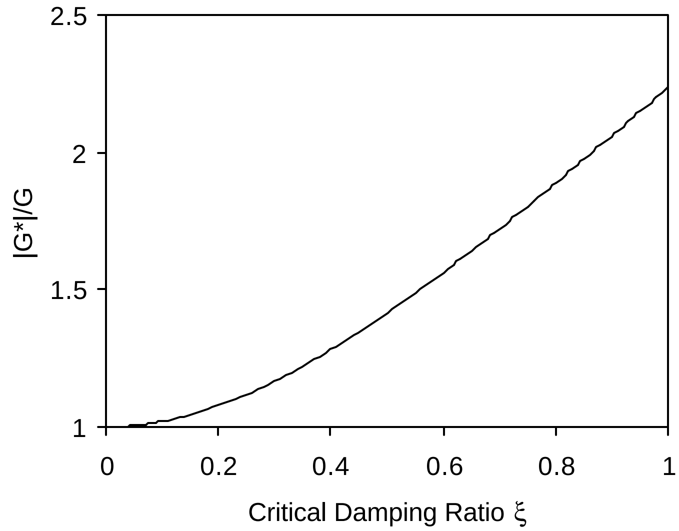


Figure 4. Normalized variation of complex shear modulus amplitude with critical damping ratio (Model 1).

2.2.2 Model 2

Model 2 is used in SHAKE 91 (Idriss and Sun, 1992). It assumes that the complex shear modulus is a function of ξ:

$$G^* = G\left\{\left(1-2\xi^2\right)+2\xi i\sqrt{1-\xi^2}\right\} \tag{18}$$

The assumption above is a constitutive assumption that pertains to the description of material behavior. It implies that the complex and real modulus have the same amplitude, i.e.:

$$\left|G^*\right| = G\left\{(1-2\xi^2)^2+4\xi^2(1-\xi^2)\right\} = G \tag{19}$$

The energy dissipated during a loading cycle is:

$$W_d = \frac{1}{2}\omega\gamma_c^2\int_t^{t+2\pi/\omega} 2G\xi\sqrt{1-\xi^2}\,dt = 2\pi G\xi\sqrt{1-\xi^2}\gamma_c^2 \tag{20}$$

Figure 5 shows the variation of dissipated energy with ξ. The dissipated energy of model 2 tends toward zero as tends toward 1. For practical purposes, ξ is usually less than 25%. Under these conditions, the energies dissipated by Models 1 and 2 are similar.

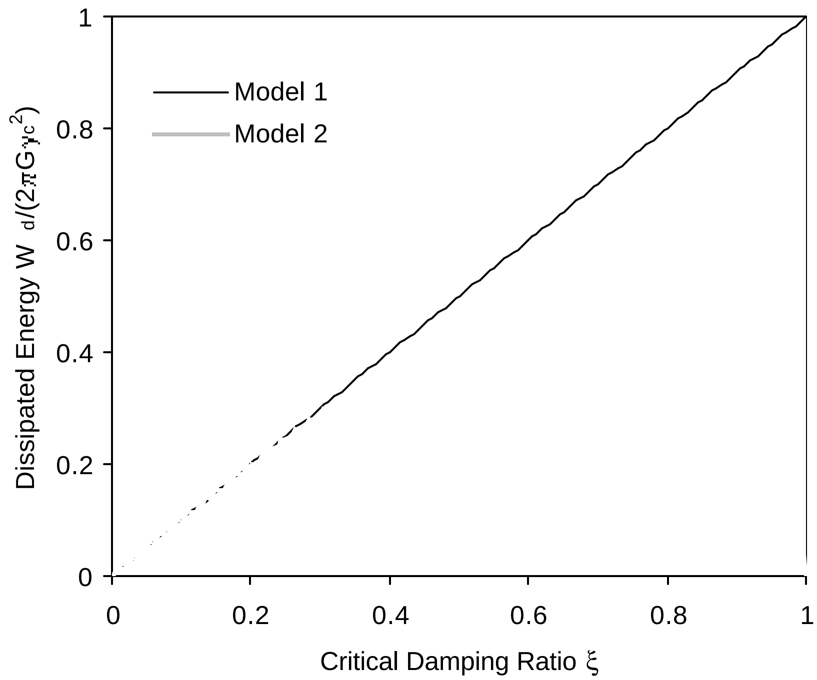


Figure 5. Normalized variation of energy dissipated per loading cycle as a function of critical damping ratio for models 1 and 2.

3. ONE-DIMENSIONAL GROUND RESPONSE ANALYSIS

Figure 6 schematizes the assumptions of one-dimensional equivalent linear site response analysis. As shown in Fig. 6, a vertical harmonic shear wave propagates in a one-dimensional layered system. The one-dimensional equation of motion for vertically propagating shear waves is:

$$\rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial \tau}{\partial z} \tag{21}$$

where r is the unit mass in any layer. Assuming that the soil in all layers behaves as a Kelvin-Voigt solid (i.e., Eq. 1), Eq. 22 becomes:

$$\rho \frac{\partial^2 u}{\partial t^2} = G \frac{\partial^2 u}{\partial z^2} + \eta \frac{\partial^3 u}{\partial z^2 \partial t} \tag{22}$$

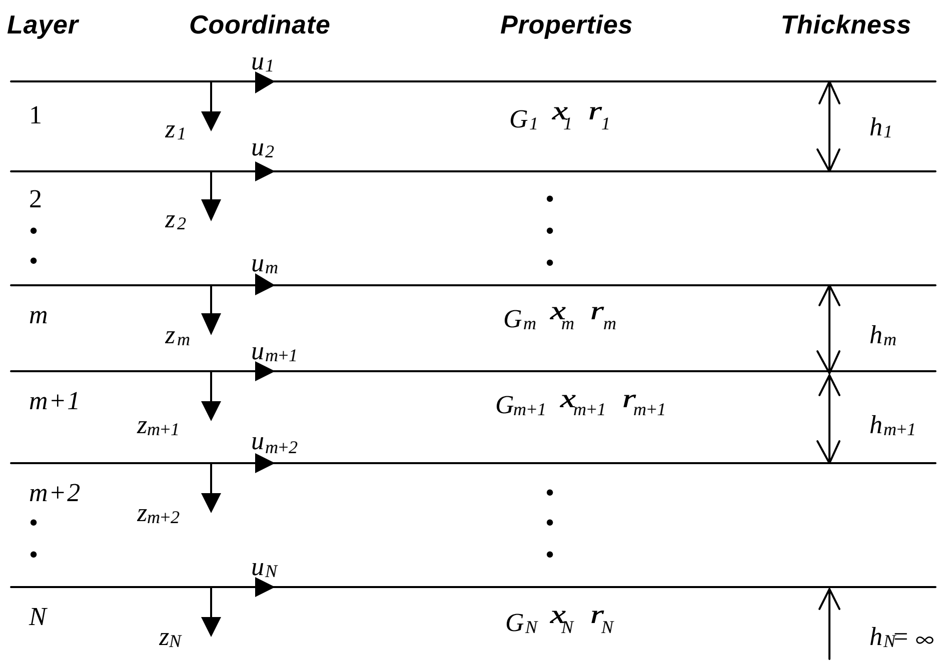


Figure 6. One-dimensional layered soil deposit system (after Schnabel et al., 1972).

For harmonic waves, the displacement can be written as:

$$u(z,t) = U(z)e^{i\omega t} \tag{23}$$

Using Eq. 23, Eq. 22 becomes:

$$(G + i\omega\eta)\frac{d^2U}{dz^2} = \rho\omega^2 U \tag{24}$$

and admits the following general solution:

$$U(x) = Ee^{ik^*z} + Fe^{-ik^*z} \tag{25}$$

where $k^{*2} = \dfrac{\rho\omega^2}{G + i\omega\eta} = \dfrac{\rho\omega^2}{G^*}$ is the complex wave number. After introducing the critical damping ratio ξ so that $\xi = \omega\eta/2G$, the complex shear modulus G^* becomes:

$$G^* = G + i\omega\eta = G(1 + 2i\xi) \tag{26}$$

The solution of Eq. 24 is:

$$u(z,t) = (Ee^{ik^*z} + Fe^{-ik^*z})e^{iwt} \tag{27}$$

and the corresponding stress is:

$$\tau(z,t) = ik^*G^*(Ee^{ik^*z} - Fe^{-ik^*z})e^{i\omega t} \tag{28}$$

The displacements at the top (z = 0) and bottom (z = h_m) of layer m of thickness h_m are:

$$u_m(0,t) = u_m = (E_m + F_m)e^{iwt} \text{ and } u_m(h_m,t) = (E_m e^{ik_m^* h_m} + F_m e^{-ik_m^* h_m})e^{iwt} \tag{29}$$

The shear stresses at the top and bottom of layer *m* are:

$$\tau_m(0,t) = ik_m^* G_m^* (E_m - F_m) e^{i\omega t} \text{ and } \tau_m(h_m,t) = ik_m^* G_m^* (E_m e^{ik_m^* h_m} - F_m e^{-ik_m^* h_m}) e^{i\omega t} \tag{30}$$

At the interface between layers *m* and *m+1*, displacements and shear stress must be continuous, which implies that:

$$u_m(h_m,t) = u_{m+1}(0,t) \text{ and } \tau_m(h_m,t) = \tau_{m+1}(0,t) \tag{31}$$

Using Eqs. 29 to 31 the coefficients E_m and F_m are related through:

$$E_{m+1} + F_{m+1} = E_m e^{ik_m^* h_m} + F_m e^{-k_m^* h_m} \tag{32}$$

$$E_{m+1} - F_{m+1} = \frac{k_m^* G_m^*}{k_{m+1}^* G_{m+1}^*} (E_m e^{ik_m^* h_m} - F_m e^{-ik_m^* h_m}) \tag{33}$$

Eqs. 32 and 33 give the following recursion formulas for amplitudes E_{m+1} and F_{m+1} in terms of E_m and F_m:

$$E_{m+1} = \frac{1}{2} E_m (1 + \alpha_m^*) e^{ik_m^* h_m} + \frac{1}{2} F_m (1 - \alpha_m^*) e^{-ik_m^* h_m} \tag{34}$$

$$F_{m+1} = \frac{1}{2} E_m (1 - \alpha_m^*) e^{ik_m^* h_m} + \frac{1}{2} F_m (1 + \alpha_m^*) e^{-ik_m^* h_m} \tag{35}$$

where α_m^* is the complex impedance ratio at the interface between layers *m* and *m+1*:

$$\alpha_m^* = \frac{k_m^* G^*_m}{k_{m+1}^* G^*_{m+1}} = \sqrt{\frac{\rho_m G^*_m}{\rho_{m+1} G^*_{m+1}}} \tag{36}$$

The recursive algorithm is started at the top free surface, for which there is no shear stress:

$$\tau_1(0,t) = ik_1^* G_1^* (E_1 - F_1) e^{i\omega t} = 0 \tag{37}$$

which implies:

$$E_1 = F_1 \tag{38}$$

Eqs. 34 and 35 are then applied successively to layers 2 to *m*. The transfer function A_{mn} relating the displacements at the top of layers *m* and *n* is defined by

$$A_{mn}(\omega) = \frac{u_m}{u_n} = \frac{E_m + F_m}{E_n + F_n} \tag{39}$$

The velocity $\dot{u}(z,t)$ and acceleration $\ddot{u}(z,t)$ are related to displacement through:

$$\dot{u}(z,t) = \frac{\partial u}{\partial t} = i\omega\, u(z,t) \quad \text{and} \quad \ddot{u}(z,t) = \frac{\partial^2 u}{\partial t^2} = -\omega^2 u(z,t) \tag{40}$$

Therefore A_{mn} is also the transfer function relating the velocities and diplacements at the top of layers *m* and *n*:

$$A_{mn}(\omega) = \frac{u_m}{u_n} = \frac{\dot{u}_m}{\dot{u}_n} = \frac{\ddot{u}_m}{\ddot{u}_n} = \frac{E_m + F_m}{E_n + F_n} \tag{41}$$

The shear strain at depth z and time t can be derived from Eq. 25:

$$\gamma(z,t) = \frac{\partial u}{\partial z} = ik^{*}(Ee^{ik^{*}z} - Fe^{-ik^{*}z})e^{i\omega t} \tag{42}$$

The corresponding shear stress at depth z and time t is:

$$\boldsymbol{t}(z,t) = G^{*}g(z,t) \tag{43}$$

3.1 Free surface, bedrock outcropping and rock outcropping motions

Figure 7 defines four terms used in site response analysis. The *free surface motion* is the motion at the surface of a soil deposit. The *bedrock motion* is the motion at the base of the soil deposit. The *rock outcropping motion* is the motion at a location where bedrock is exposed at the ground surface.

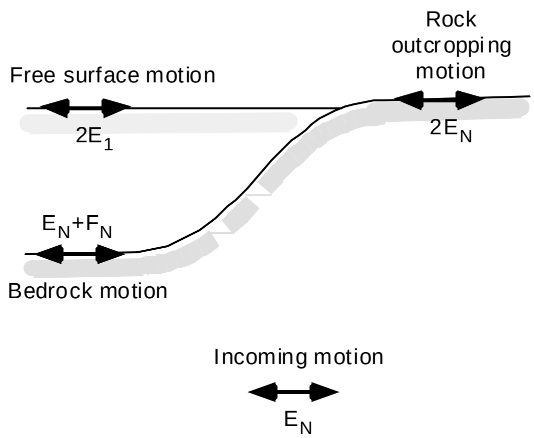


Figure 7. Terminology used in site response analysis.

As shown in Fig. 7, the incoming shear wave that propagates vertically upward has for amplitude E_N through the bedrock. The bedrock motion has for amplitude E_N+F_N at the top of the bedrock under the soil layers. The bedrock outcropping motion is $2E_N$, because there is no shear stress (i.e., $E_N = F_N$) on free surfaces. Therefore the transfer function relating the bedrock motion and bedrock outcropping motion is:

$$A_{NN}(\boldsymbol{w}) = \frac{2E_N}{E_N + F_N} \tag{44}$$

When it is assumed that $E_1 = F_1 = 1$ on free surface, then the transfer function relating the free surface motion and rock outcropping motion is:

$$A'_{1N}(\boldsymbol{w}) = \frac{1}{E_N} \tag{45}$$

3.2 Transient Motions

The theory above for the one-dimensional soil column response was presented for steady state harmonic motions, i.e., in the frequency domain. It can be extended to the time histories of transient motions using Fourier series (e.g., Bendat and Piersol, 1986). A real-valued or complex-valued function $x(t)$ can be approximated by a discrete series of N values as follows:

$$x_n = \sum_{k=0}^{N-1} X_k e^{iw_k t_n} = \sum_{k=0}^{N-1} X_k e^{iw_k n\Delta t} = \sum_{k=0}^{N-1} X_k e^{2pikn/N} \quad n = 0, \ldots, N-1 \tag{46}$$

The values x_n correspond to times $t_n = n\ Dt$ where Dt is the constant time interval (i.e., $x(nDt) = x_n$ for n = 0, …, N-1). The discrete frequencies w_k are:

$$w_k = 2p \frac{k}{N\ \Delta t} \qquad k = 0, \ldots, N\text{-}1 \tag{47}$$

The Fourier components are:

$$X_m = \frac{1}{N}\sum_{k=0}^{N-1} x_n e^{-2pikm/N} \qquad m = 0, \ldots, N\text{-}1 \tag{48}$$

The coefficients X_m are calculated by the Fast Fourier Transform algorithm, which was originally developed by Cooley and Tukey (1965). The number of operations scales as $N\ logN$, which justifies the name of Fast Fourier Transform (i.e., FFT).

3.3 Iterative Approximation of Equivalent Linear Response

As previously described in Fig. 3, the equivalent linear model assumes that the shear modulus and damping ratio are functions of shear strain amplitude. In SHAKE, the values of shear modulus and damping ratio are determined by iterations so that they become consistent with the level of strain induced in each layer. As shown in Fig. 8, the values of G_0 and x_0 are initialized at their small strain values, and the maximum shear strain g_{max} and effective shear strain g_{eff1} are calculated. Then the compatible values G_1 and x_1 corresponding to g^1_{eff} are found for the next iteration. The equivalent linear analysis is repeated with new values of G and x until the values of G and x are compatible with the strain induced in all layers.

The iteration procedure for equivalent linear approach in each layer is as follows:

1. Initialize the values of G^i and x^i at their small strain values.
2. Compute the ground response, and get the amplitudes of maximum shear strain g_{max} from the time histories of shear strain in each layer.
3. Determine the effective shear strain g_{eff} from γ_{max}:

$$\gamma^i_{eff} = R_\gamma \gamma^i_{\max} \tag{49}$$

   where R_g is the ratio of the effective shear strain to maximum shear strain, which depends on the earthquake magnitude. R_g is specified in input; it accounts for the number of cycles during earthquakes. R_g is the same for all layers.
4. Calculate the new equivalent linear values G_{i+1} and x_{i+1} corresponding to the effective shear strain g_{eff}.
5. Repeat steps 2 to 4 until the differences between the computed values of shear modulus and damping ratio in two successive iterations fall below some predetermined value in all layers. Generally 8 iterations are sufficient to achieve convergence.

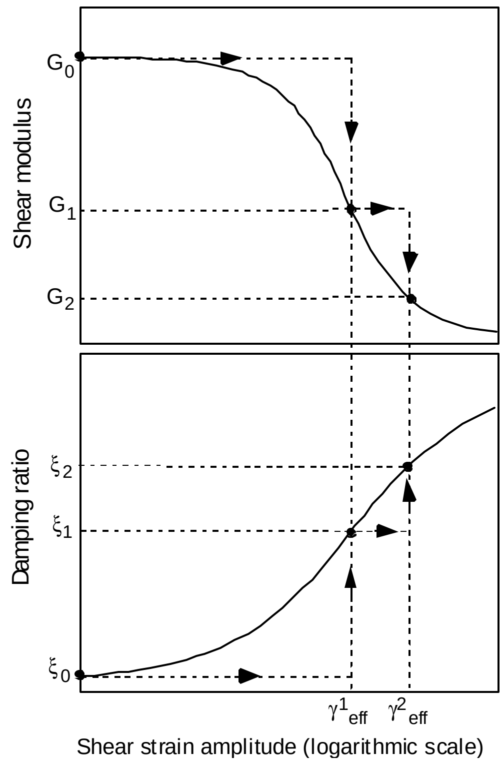


Figure 8. Iteration of shear modulus and damping ratio with shear strain in equivalent linear analysis.

4. DESCRIPTION OF EERA

EERA (Equivalent-linear Earthquake site Response Analysis) is a modern implementation of the equivalent-linear concept of earthquake site response analysis, which was previously implemented in the original and subsequent versions of SHAKE (Schnabel et al., 1972; and Idriss and Sun, 1991).

4.1 System requirement, distribution files and download EERA

EERA requires Windows 95/98/NT and EXCEL 97 or later. EERA does not work for other operating systems and spreadsheet program. All the files required for EERA can be downloaded from http://geoinfo.usc.edu/gees in one zip file named EERA.zip, which contains the following files:

- EERA.xla (389 kB), Microsoft Excel Add-in
- EERAM.xls (4627 kB), Example Startup file (metric units)
- EERA.xls (3988kB), Example Startup file (British units)
- EERA.dll (97kB), EERA Dynamic Link Library
- Diam.acc (21kB), Example earthquake input acceleration file

4.2 Installing and removing EERA

It is recommended to install EERA on your computer system as follows:

1. Copy the distribution files in a new directory on your hard drive
2. Move EERA.dll and EERA.xla in the same directory of your choice. This is a critical step as these files cannot be moved easily later.
3. In EXCEL, install the Add-in EERA.xla using ***Tools*** and ***Add-ins...*** As shown in Fig. 9, use ***Browse*** to locate EERA.xla. Do not move EERA.xla after installing it.

4. When EERA is properly installed, the EERA menu will appear to the right of the EXCEL pull-down menus (Fig. 10).

EERA can be de-installed by using ***Remove EERA*** in the EERA pull-down menu.

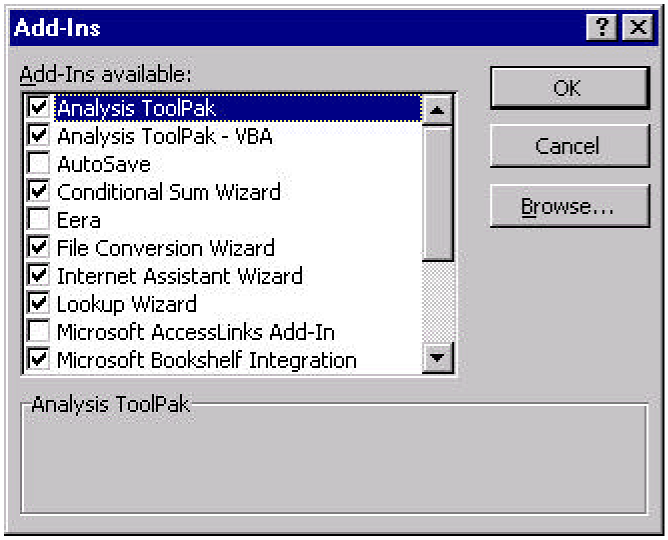


Figure 9. The EXCEL Add-ins menu.

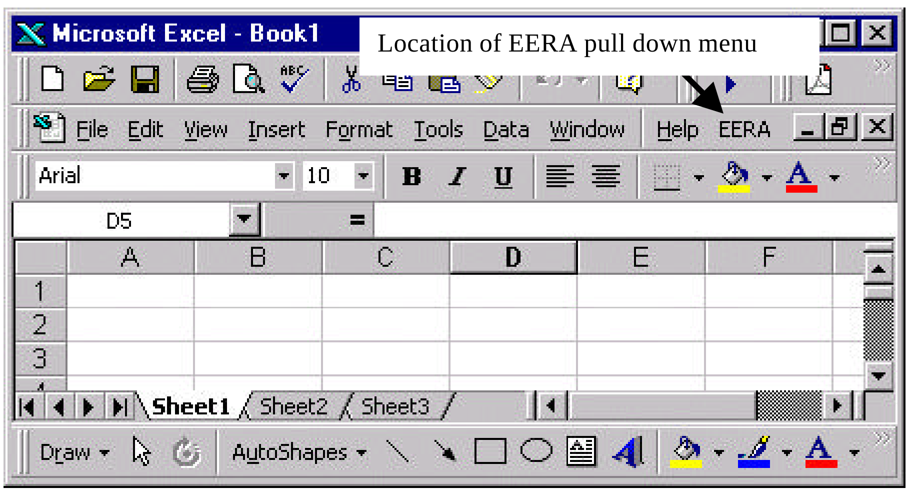


Figure 10. After a successful installation of EERA, the pull-down menu EERA should appear to the right of EXCEL pull down menus.

4.3 EERA commands

As shown in Fig. 11, there are seven commands in the EERA pull-down menu

1. ***Process Earthquake Data*** - Read and process earthquake input motion (input/output in worksheet *Earthquake*)
2. ***Calculate Compatible Strain*** - Read profile, material curves, and execute the main iterative calculation (input/output in worksheet *Iteration*)
3. ***Calculate Output***
   - ***Acceleration/Velocity/Displacement*** - Calculate time history of acceleration, relative velocity and displacement at the top of selected sub-layers (input/output in worksheet *Acceleration ...*)
   - ***Stress/Strain*** - Calculate stress and strain at the middle of selected sub-layers (input/output in worksheets *Strain ...*)

- ***Amplification*** - Calculate amplification factors between two sub-layers (input/output in worksheets *Ampli ...*)
- ***Fourier Spectrum*** - Calculate Fourier amplitude spectrum of acceleration at the top of selected sub-layer. (input/output in worksheet *Fourier ...*)
- ***Response Spectrum*** - Calculate all response spectra at the top of selected sub-layers (input/output in worksheet *Spectra ...*)
- ***All of the above*** - Calculate all the output

4. ***Duplicate Worksheet*** - Duplicate selected worksheet for defining new material curves, and adding new output (e.g., response spectra for several sub-layers)
5. ***Delete Worksheet*** - Delete unnecessary worksheet (some worksheet cannot be deleted)
6. ***Remove EERA*** - De-install EERA from EXCEL
7. ***About EERA*** - Number of EERA version

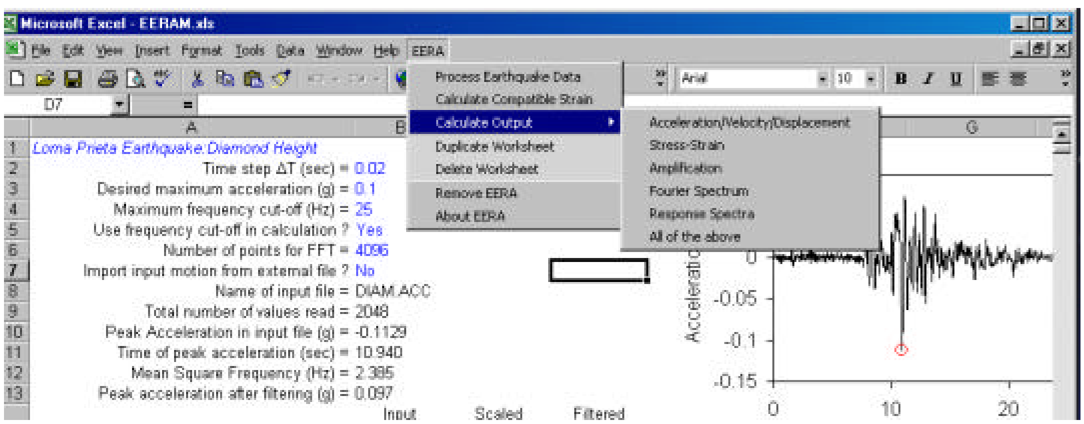


Figure 11. The EERA pull-down menu.

EERA commands are to be used in the following order:

1. ***Process Earthquake Data***
2. ***Calculate Compatible Strain***
3. ***Calculate Output***

4.4 EERA worksheets

As shown in Table 1 and Figs. 12 to 19, an EERA workbook is made of nine types of worksheets, which have predefined names that should not be changed. As indicated in Table 1, six of nine types of worksheet can be duplicated and modified using ***Duplicate Worksheet*** in the EERA pull-down menu. This feature is useful for obtaining output at several sub-layers and defining additional material curves. Table 1 also indicates the number of input required in each worksheet.

Table 1. Types of worksheets in EERA and their contents

Worksheet	Contents	Duplication	Number of input
Earthquake	Earthquake input time history	No	7
Mat I	Material curves (G/G_{max} and Damping versus strain for material type i	Yes	Dependent on number of soil layers
Profile	Vertical profile of layers	No	Dependent on number of data points per material curve
Iteration	Results of main calculation	No	3
Acceleration	Time history of acceleration/velocity/displacement	Yes	2
Strain	Time history of stress and strain	Yes	1
Ampli	Amplification between two sub-layers	Yes	4
Fourier	Fourier amplitude spectrum of acceleration	Yes	3
Spectra	Response spectra	Yes	3

4.4.1 Earthquake data

As shown in Fig. 12, Worksheet *Earthquake* is used to define the earthquake input motion. There are six required entries and one optional entry. All entries are in blue characters.

- *Cell A1*: the earthquake name is optional.
- *Cell B2*: the time step DT is the time interval between the evenly spaced data points of the time history of input ground motion.
- *Cell B3*: the desired maximum frequency is used to scale the peak amplitude of the input acceleration.
- *Cell B4*: the maximum frequency cut-off f_{max} is used to filter the high frequencies from the input acceleration.
- *Cell B5*: the frequency cut-off f_{max} can be used to eliminate high frequencies from the input acceleration records. All the calculations will be performed for frequencies between 0 and f_{max}. This option is useful to overcome the overflow calculation errors in Eq. 34 that are usually caused for very high frequencies.
- *Cell B6*: the number m of data points in the FFT calculation can be defined. m is generally selected to be larger than the number n of data points in the input acceleration time history. In this case the input record is padded with zero in order to produce a record of length n.
- *Cell B7*: The input acceleration can be read from an external data file. EERA is capable of reading many earthquake data formats from external data files. In this case, select ***Yes*** in Cell B5, then follow EERA instructions for selecting data format. As shown in Fig. 13, you can open earthquake files of different types, eliminate headers, and select various formats. Header lines are skipped by selecting a starting row number in Fig. 13. If you do not wish to import earthquake data for an external file, you may select ***No*** in Cell B5. This option is useful when (1) you have already imported earthquake data, or (2) you have pasted your own earthquake acceleration time history in column B15-.

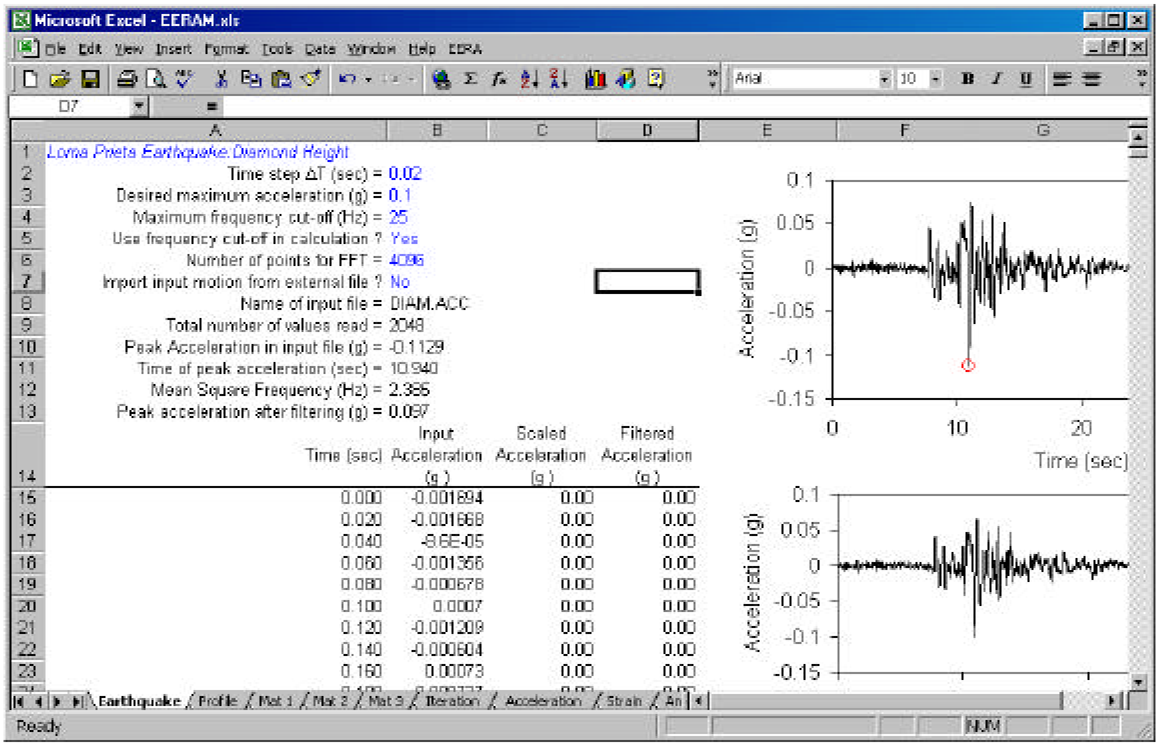


Figure 12. Worksheet *Earthquake*.

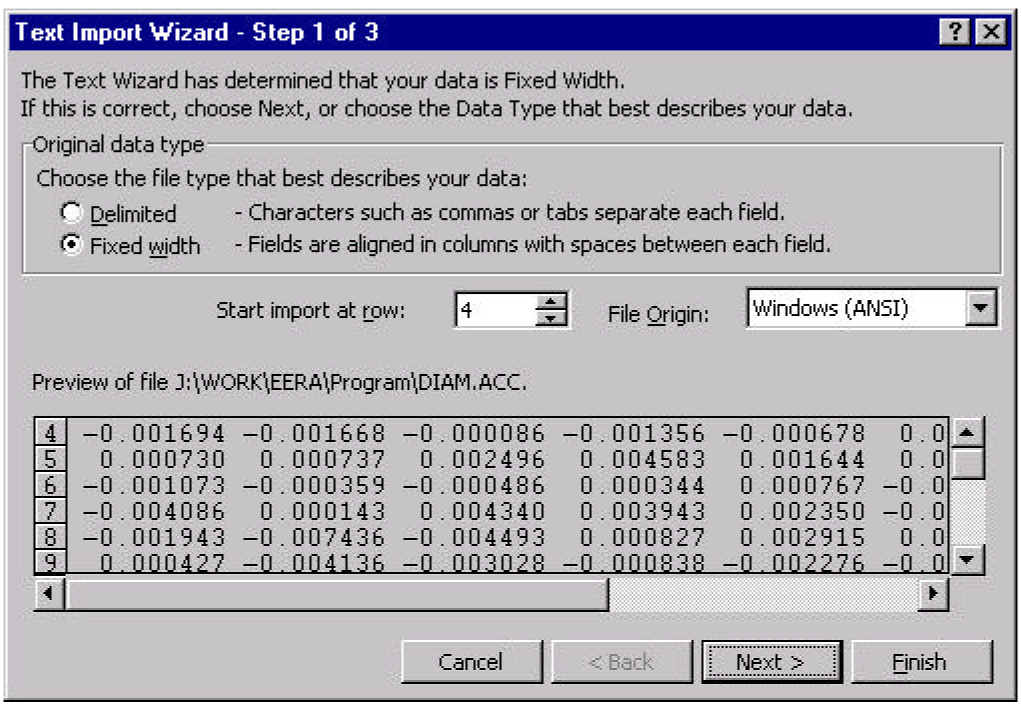


Figure 13. Importing earthquake input data.

4.4.2 Soil Profile

As shown in Fig. 14, Worksheet *Profile* is used to define the geometry and properties of the soil profile. All input data are in blue cells. Input data can be graphically checked as shown in Fig. 15.

- *Cell A1*: the soil profile is optionally named.

- *Column C6-* : the number of material type is specified for each layer. Each material type i is defined in a separate worksheet called ***Mat i***.
- *Column D6-* : each layer may be subdivided in several sub-layers. This feature improves the accuracy of calculations.
- *Column E6-* : the thickness of each layer is specified.
- *Column F6-* : The small strain values of shear modulus is entered in the unit specified in Cell F5. If this column is left blank then the shear wave velocity must be input in column I6-
- *Column G6-* : The initial value of critical damping is only required when the material number on this row is equal to zero, (i.e., no material curve is defined).
- *Column H6-* : The total unit weight is entered in the physical unit specified in Cell H5
- *Column I6-* : The shear wave velocity is enter in the physical unit specified in Cell I5. If this column is left blank then the maximum shear modulus must be input in column F6-
- *Column J6-* : The location and type of earthquake motion is defined by specifying only once in this column either ***Outcrop*** for an outcropping rock motion, or ***Inside*** for a non outcropping motion.
- *Column K6-* : The depth of the water table can be specified in order to calculate vertical effective stresses. This input is optional as it is only used in the calculation of initial stresses, and not in other calculations.

- *Cell E3*: The average shear wave velocity *V* of the soil profile is calculated as follows:

  $$V = \frac{1}{\sum_{i=1}^{N} h_i} \sum_{i=1}^{N} h_i v_i$$

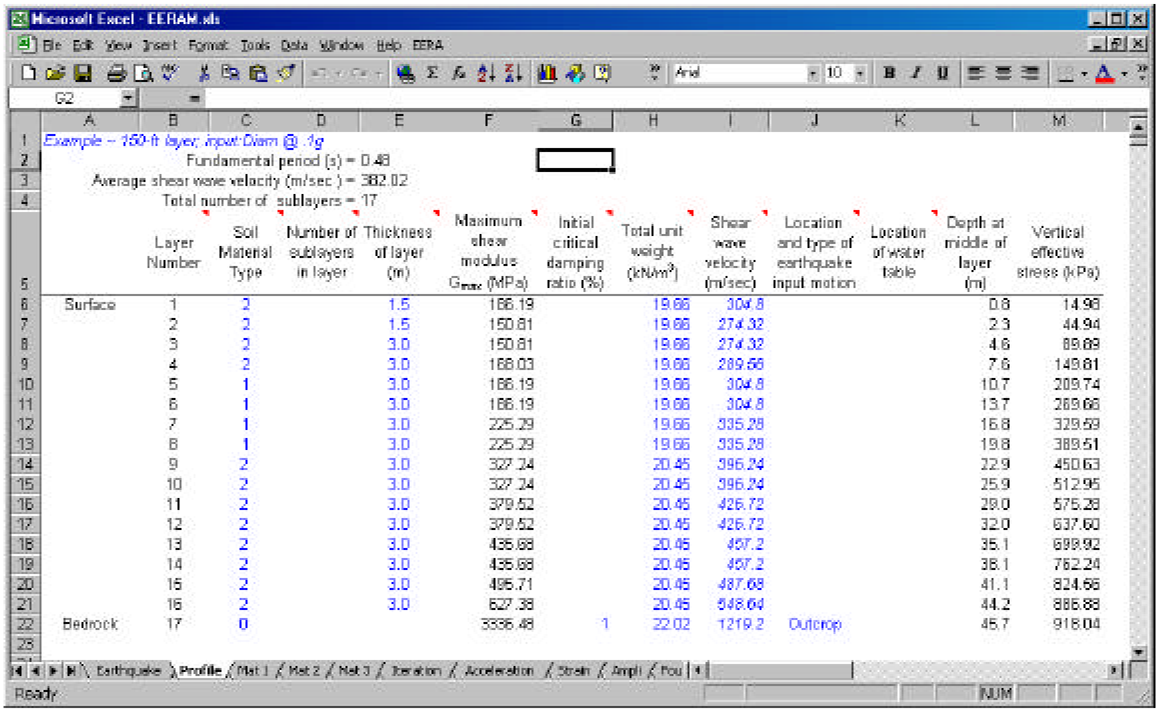
  where h_i is the height of layer i, v_i is the shear wave velocity in layer i, and N is the total number of layers.

- *Cell E2*: The fundamental period *T* of the soil profile is calculated as *T = 4 H/V* where *H* is the total thickness of soil profile and *V* is the average shear wave velocity of soil profile as calculated in Cell E2.

Note:

The average shear wave velocity *V* and fundamental period *T* can also be computed as follows

$$V = \frac{\sum_{i=1}^{N} h_i}{\sum_{i=1}^{N} \frac{h_i}{v_i}} \quad \text{and} \quad T = 4\sum_{i=1}^{N} \frac{h_i}{v_i}$$



Example – 150-ft layer; input:Diam @ .1g

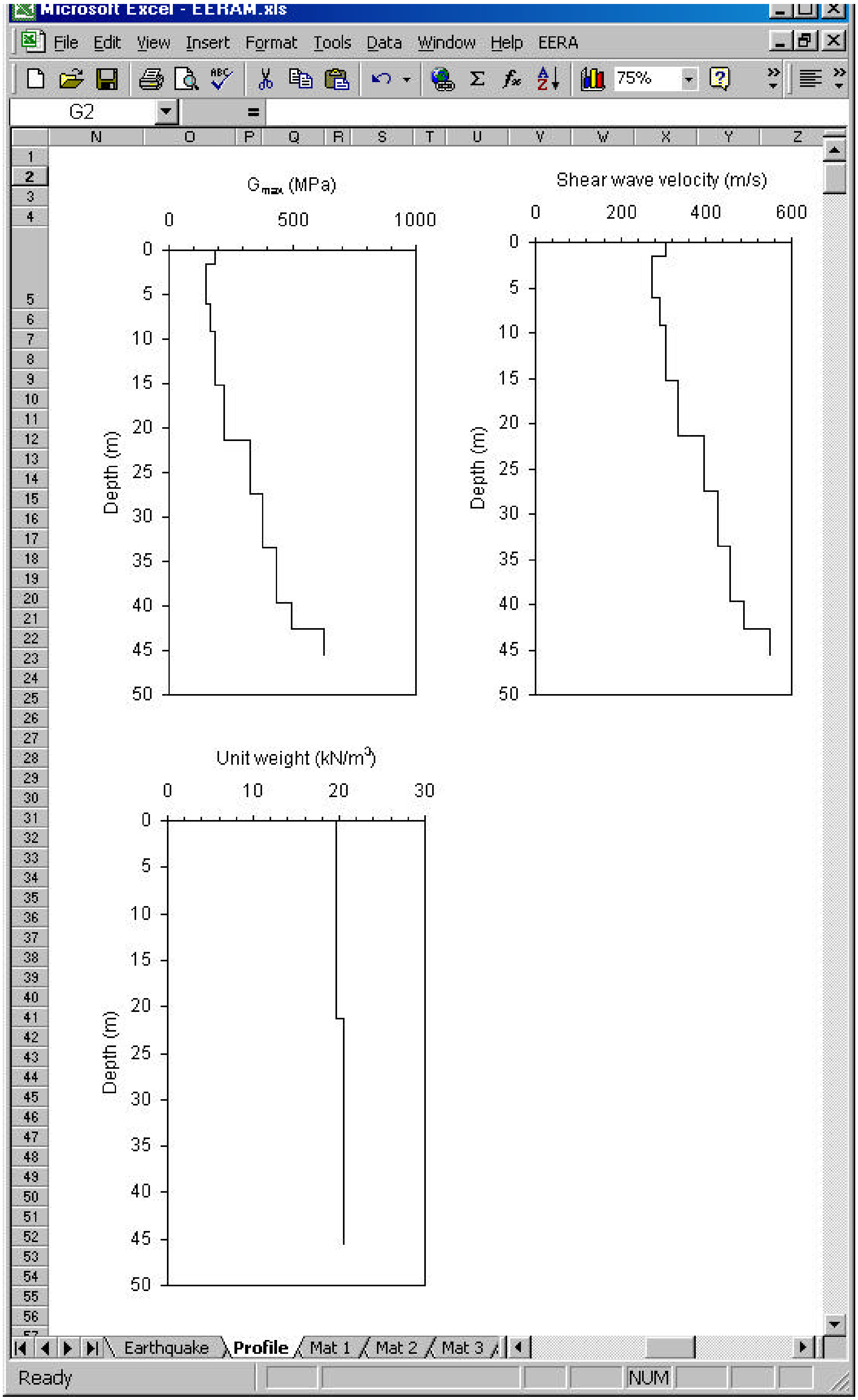
Fundamental period (s) = 0.48

Average shear wave velocity (m/sec) = 382.02

Total number of sublayers = 17

	Layer Number	Soil Material Type	Number of sublayers in layer	Thickness of layer (m)	Maximum shear modulus G_{max} (MPa)	Initial critical damping ratio (%)	Total unit weight (kN/m^3)	Shear wave velocity (m/sec)	Location and type of earthquake input motion	Location of water table	Depth at middle of layer (m)	Vertical effective stress (kPa)
Surface	1	2		1.5	186.19		19.66	304.8			0.8	14.98
	2	2		1.5	150.81		19.66	274.32			2.3	44.94
	3	2		3.0	150.81		19.66	274.32			4.6	89.89
	4	2		3.0	168.03		19.66	289.56			7.6	149.81
	5	1		3.0	186.19		19.66	304.8			10.7	209.74
	6	1		3.0	186.19		19.66	304.8			13.7	269.66
	7	1		3.0	225.29		19.66	335.28			16.8	329.59
	8	1		3.0	225.29		19.66	335.28			19.8	389.51
	9	2		3.0	327.24		20.45	396.24			22.9	450.63
	10	2		3.0	327.24		20.45	396.24			25.9	512.95
	11	2		3.0	379.52		20.45	426.72			29.0	575.28
	12	2		3.0	379.52		20.45	426.72			32.0	637.60
	13	2		3.0	435.68		20.45	457.2			35.1	699.92
	14	2		3.0	435.68		20.45	457.2			38.1	762.24
	15	2		3.0	495.71		20.45	487.68			41.1	824.56
	16	2		3.0	627.38		20.45	548.64			44.2	886.88
Bedrock	17	0			3336.48	1	22.02	1219.2	Outcrop		45.7	918.04

Figure 14. Worksheet *Profile*.



Microsoft Excel - EERAM.xls
File Edit View Insert Format Tools Data Window Help EERA
75%
G2
=
Gmax (MPa)
0 500 1000
Depth (m)
Shear wave velocity (m/s)
0 200 400 600
Depth (m)
Unit weight (kN/m3)
0 10 20 30
Depth (m)
Earthquake
Profile
Mat 1
Mat 2
Mat 3
Ready
NUM

Figure 15. Worksheet *Profile*.

4.4.3 Material stress-strain damping-strain curves

As shown in Fig. 16, several material stress-strain and damping-strain curves can be defined. You may generate additional worksheets for material properties by using ***Duplicate worksheet*** from the main EERA menu. Input data can be graphically checked as shown in Fig. 16.

- *Cell A1*: the material type is optionally named.
- *Column A3-* : the values of shear strain corresponding to ratio G/G_{max} data in column B3- are entered as increasing numbers.
- *Column B3-* : Enter the values of ratio G/G_{max} corresponding to strain data in column A3-.
- *Column C3-* : the values of shear strain corresponding to critical damping ratio data in column D3- are entered as increasing numbers.
- *Column D3-* : Enter the values of critical damping ratio corresponding to strain data in column C3-.

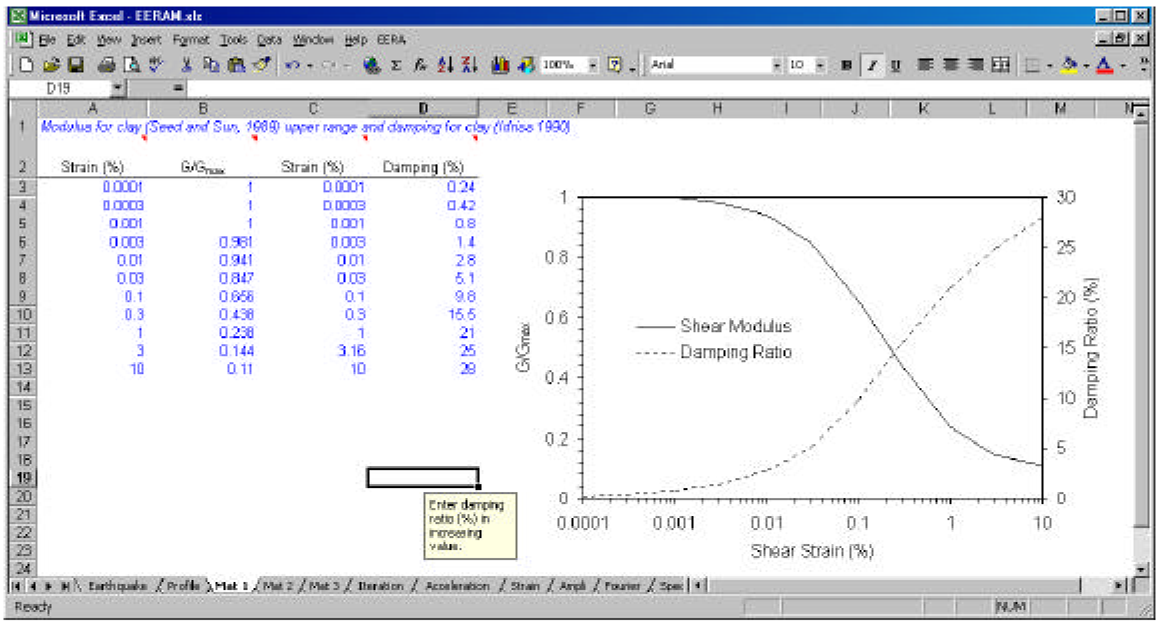


Figure 16. Worksheet *Mat*.

4.4.4 Calculation

As shown in Fig. 17, the worksheet iteration has three entries (shown in blue characters):

- *Cell E1*: the number of iterations is specified. Eight iterations are usually enough to achieve a satisfactory convergence.
- *Column E2*: The ratio of equivalent uniform strain is entered. The ratio of equivalent uniform strain accounts for the effects of earthquake duration. Typically this ratio ranges from 0.4 to 0.75 depending on the input motion and which magnitude earthquake it is intended to represent. The following equation may be used to estimate this ratio (Idriss and Sun, 1992):
  ratio = (M - l)/10
  in which M is the magnitude of the earthquake. For example, for M = 5, the ratio would be 0.4.
- *Column E3*: the type of linear equivalent model is selected. There are two options: (1) model of the original SHAKE, and (2) model of SHAKE91.

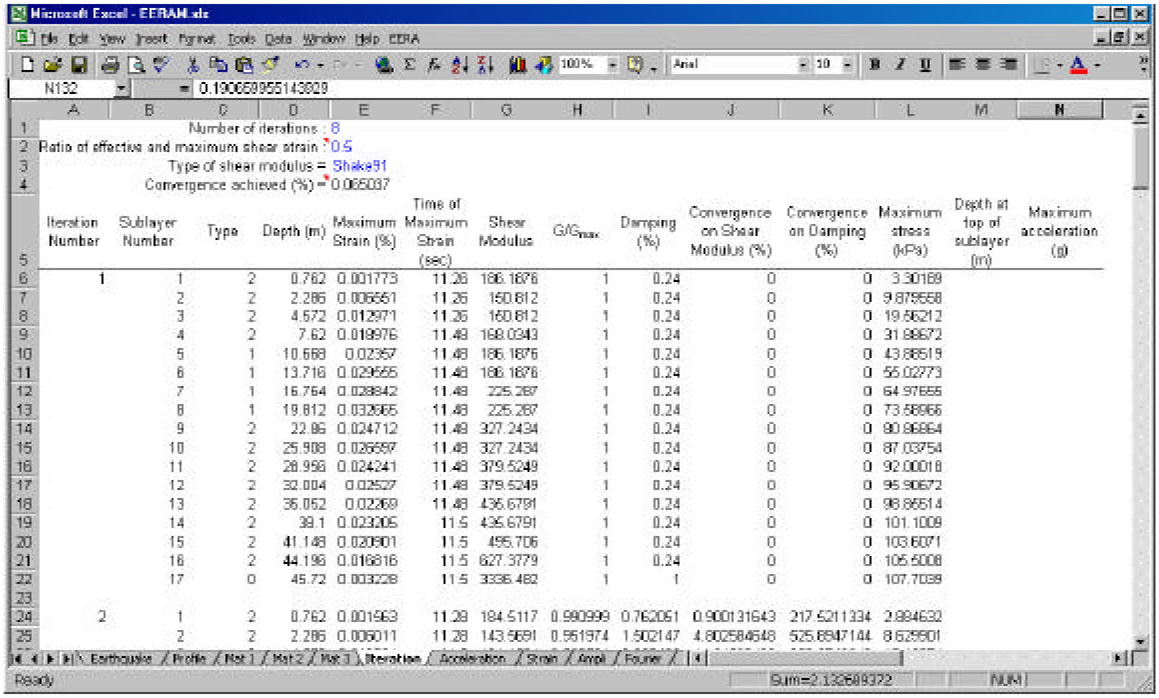


Figure 17. Worksheet *Iteration*.

4.4.5 Output (Acceleration)

As shown in Fig. 18, the worksheet Acceleration defines the time history of acceleration/ relative velocity and relative displacement at a selected sublayer. The worksheet can be duplicated by using ***Duplicate Worksheet*** in the EERA menu.

- *Cell D1*: The number of selected sublayer is specified.
- *Cell D2*: The type of selected sublayer is specified. The type can be either outcropping or not outcropping.
-

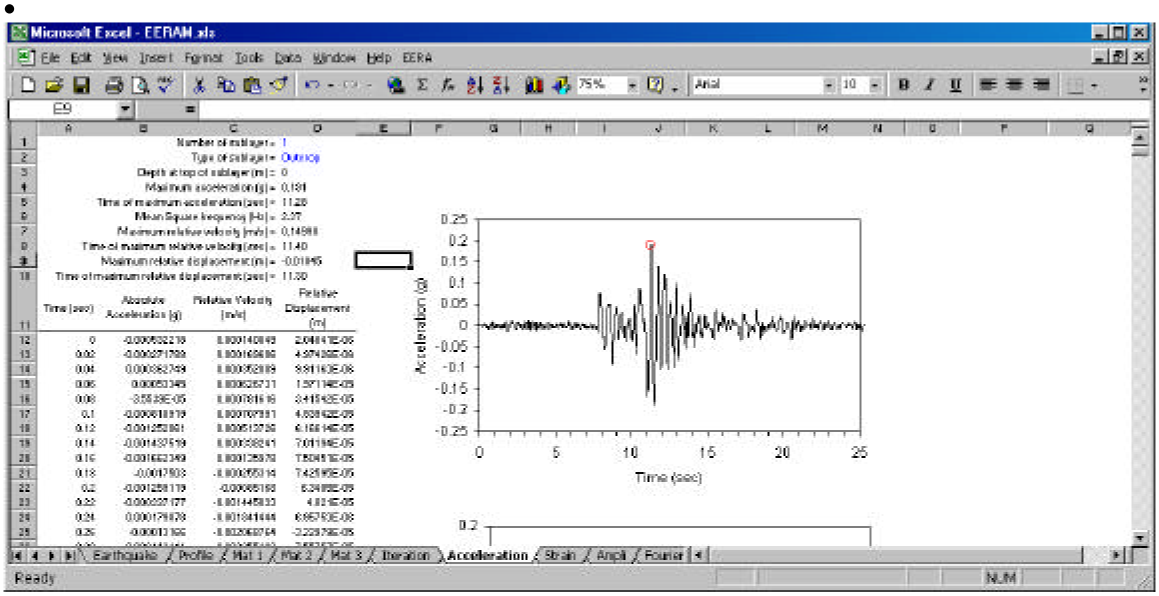


Figure 18. Worksheet *Acceleration*.

4.4.6 Output (Strain)

As shown in Fig. 19, the worksheet ***Strain*** defines the time history of stress, strain and dissipated energy, and stress-strain loops. The worksheet can be duplicated by using ***Duplicate Worksheet*** in the EERA menu.

- *Cell D1*: The number of selected sublayer is specified.
-

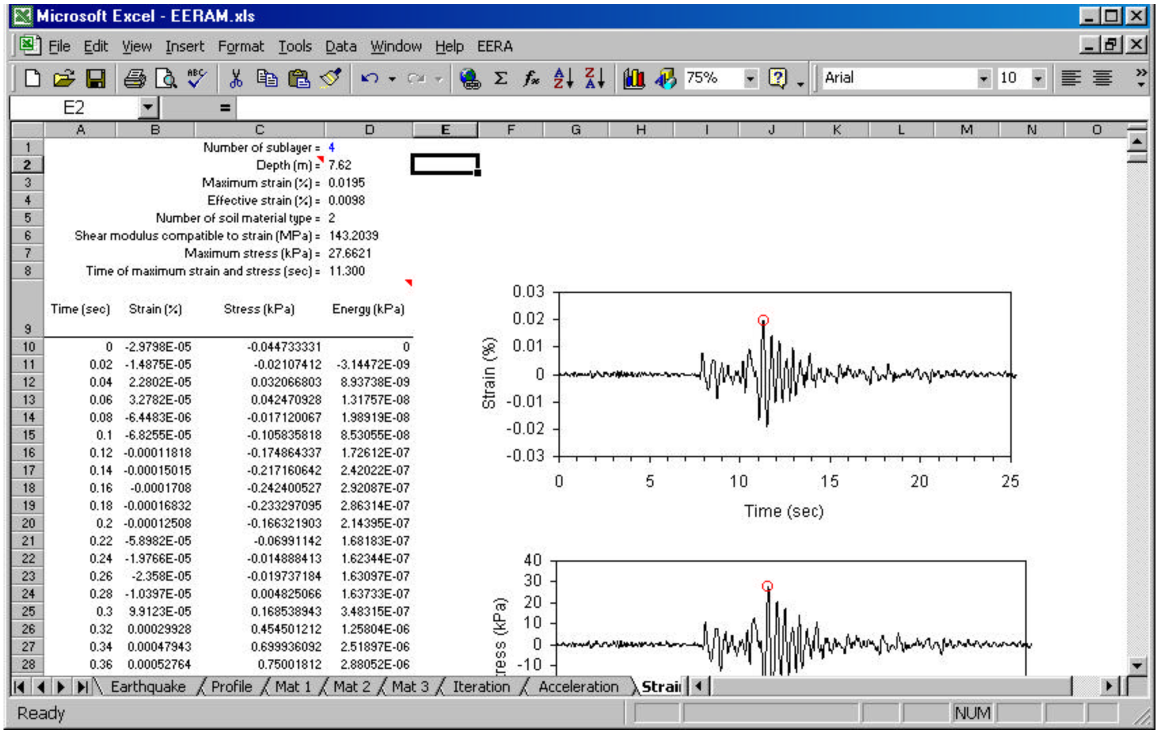


Figure 19. Worksheet *Strain*.

4.4.7 Output (Ampli)

As shown in Fig. 20, the worksheet ***Ampli*** defines the amplification factor between two sublayers. The worksheet can be duplicated by using ***Duplicate Worksheet*** in the EERA menu.

- *Cell D1*: The number of the first sublayer is specified.
- *Cell D2*: The type of the first sublayer is specified. It can be either outcropping or not outcropping.
- *Cell D3*: The number of the second sublayer is specified.
- *Cell D4*: The type of the second sublayer is specified. It can be either outcropping or not outcropping.
-

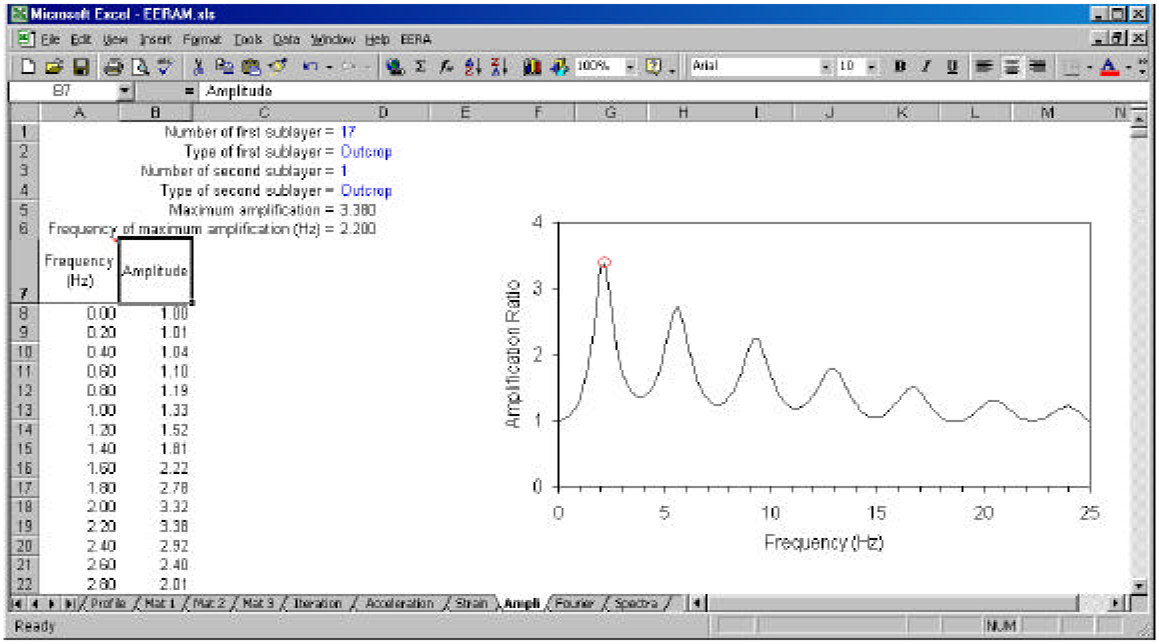


Figure 20. Worksheet *Ampli*.

4.4.8 Output (Fourier)

As shown in Fig. 21, the worksheet ***Fourier*** defines the Fourier spectrum for a selected sublayer. The worksheet can be duplicated by using ***Duplicate Worksheet*** in the EERA menu.

- *Cell D1*: The number of the selected sublayer is specified.
- *Cell D2*: The type of the first sublayer is specified. It can be either outcropping or not outcropping.
- *Cell D3*: The number of moving averages is specified. This feature filtered noises in the Fourier spectrum.
-

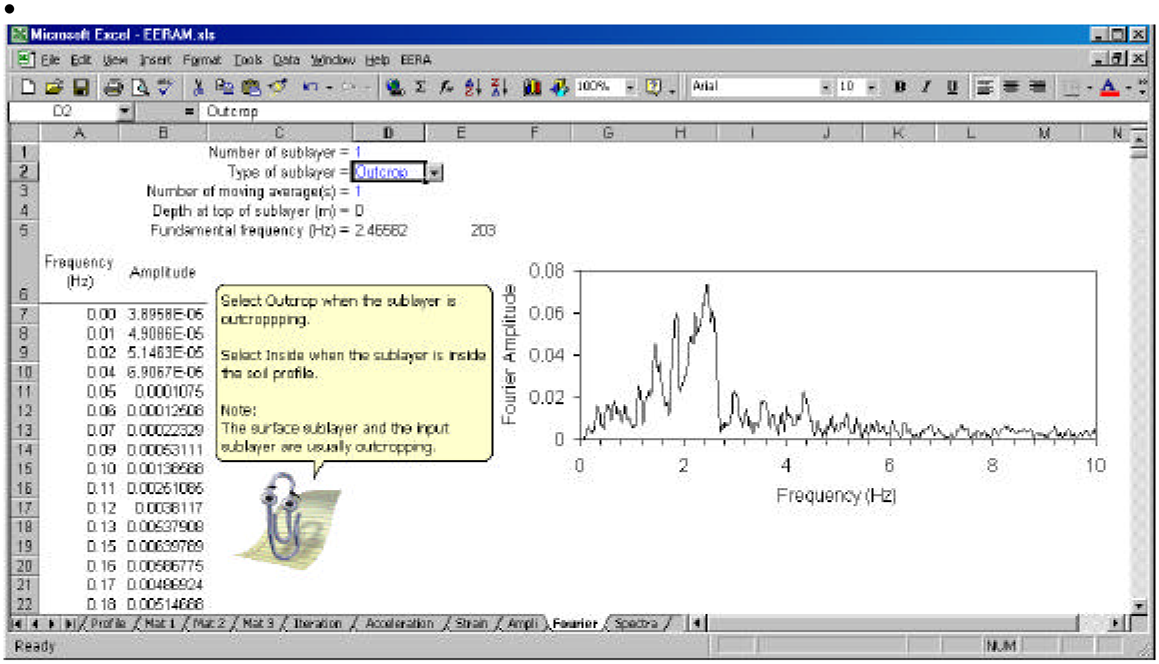


Figure 21. Worksheet *Fourier*.

4.4.9 Output (Spectra)

As shown in Fig. 22, the worksheet ***Spectra*** defines response spectra for a selected sublayer. The worksheet can be duplicated by using ***Duplicate Worksheet*** in the EERA menu.

- *Cell D1*: The number of the selected sublayer is specified.
- *Cell D2*: The type of the first sublayer is specified. It can be either outcropping or not outcropping.
- *Cell D3*: The selected value of critical damping ratio for the response spectra.

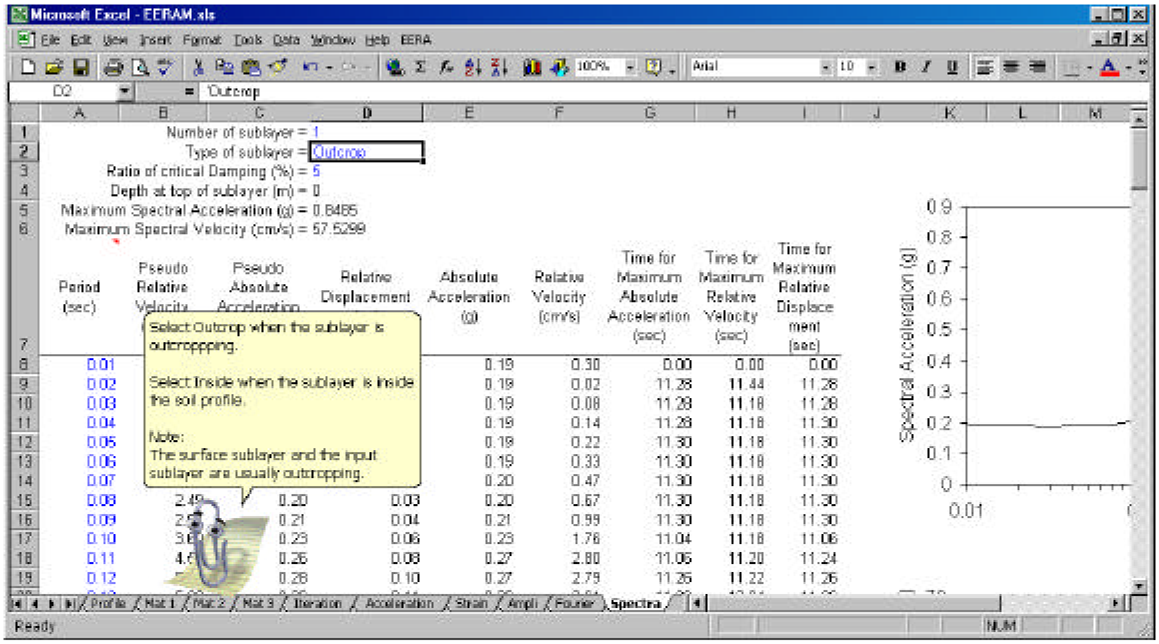


Figure 21. Worksheet *Spectra*.

4.5 Running EERA

EERA is distributed with the example file EERAM.xls. Once you have opened this example file from within EXCEL, you can perform the following operations using the EERA pull-down menu:

1. ***Process Earthquake Data***
2. ***Calculate Compatible Strain***
3. ***Calculate Output*** and ***All of the Above***

As you perform these steps above, the output data will be first erased and then recalculated. No user-input in the worksheet is required in this example. Some of the output figures are displayed in Appendix A.

You may attempt to define your own problem. EERA input data is in the cells with blue characters. A help message is displayed as you move the cursor to input cells. A suggested way to proceed is as follows:

1. Copy EERAM.xls (or an existing EERA file), and rename it as you like (e.g. Myrun.xls)
2. In Myrun.xls, retype on existing blue cells to enter your data. You may use additional line in worksheets *Mat...*, *Profile* and *Earthquake*
3. If additional material curves or output is required, duplicate worksheets using ***Duplicate Worksheet***
4. Run EERA by using successively
   1. ***Process Earthquake Data***
   2. ***Calculate Compatible Strain***
   3. ***Calculate Output*** and ***All of the Above***

In general, a EERA site response analysis is performed in three successive steps.

Step 1

- Define all earthquake data in worksheet *Earthquake*
- Use ***Process Earthquake Data***

Step 2

- Define the soil profile in worksheet *Profile*
- Define all the material stress-strain response curves in worksheets *Mat...*
- Define the main calculation parameters in worksheet *Iteration*
- Use ***Calculate Compatible Strain***

Step 3

- Define the input parameters in worksheets *Acceleration*
- Use ***Calculate Output*** and ***Acceleration/...***
- Define the input parameters in worksheets *Strain*
- Use ***Calculate Output*** and ***Stress-Strain***
- Repeat the same process for *Ampli, Fourier*, and *Spectra*

4.6 Main features of EERA and comparison with SHAKE

EERA and SHAKE91 are based on the same fundamental principles. However their implementations are substantially different. The main differences are:

- EERA dynamic library is implemented in FORTRAN 90. All matrix and vector calculations are performed without indices. One of the main advantages of FORTRAN 90 over its predecessors is the dynamic dimensioning of arrays. The size of the arrays adapt to the size of the problems, within the limit of the computer available memory. EERA dimensions internally its work arrays depending on the problem size.
- As a benefit of dynamic dimensioning, there is no limit on the total number of material properties and soil layers in EERA. The sublayers and soil properties are no longer limited to 50 and 13, respectively, as in SHAKE91.
- As another benefit of dynamic dimensioning, the users can prescribe the number of data points for FFT beyond 4096. Larger numbers of data points are useful to describe high frequencies in the calculations. However, longer computation times are likely to result from larger number of data points in the FFT calculations.
- The relative displacement and velocity can be calculated in sublayers, in the same way as acceleration.
- The users can select calculations for a filtered or unfiltered object motion. SHAKE91 uses an optional low-pass filter, which eliminates the high frequency response. The high frequency components may be useful for the users interested in dynamic response of structures with high frequency components (e.g., structural equipment).
- EERA uses an optimized version (IMSL, 1998) of the Cooley and Tukey algorithm for FFT.

4.7 Comparison of results from EERA and SHAKE

The results of EERA and SHAKE91 were found to be identical in the example provided with SHAKE91. This comparison is detailed in Appendix B.

5. CONCLUSION

EERA is a modern implementation of the equivalent linear concept for earthquake site response analysis, which was previously implemented in SHAKE. EERA is fully integrated with a spreadsheet program EXCEL and gives many new additional features to users, such as unlimited number of soil properties and soil layers, and user-defined number of data points for fast Fourier transform.

6. REFERENCES

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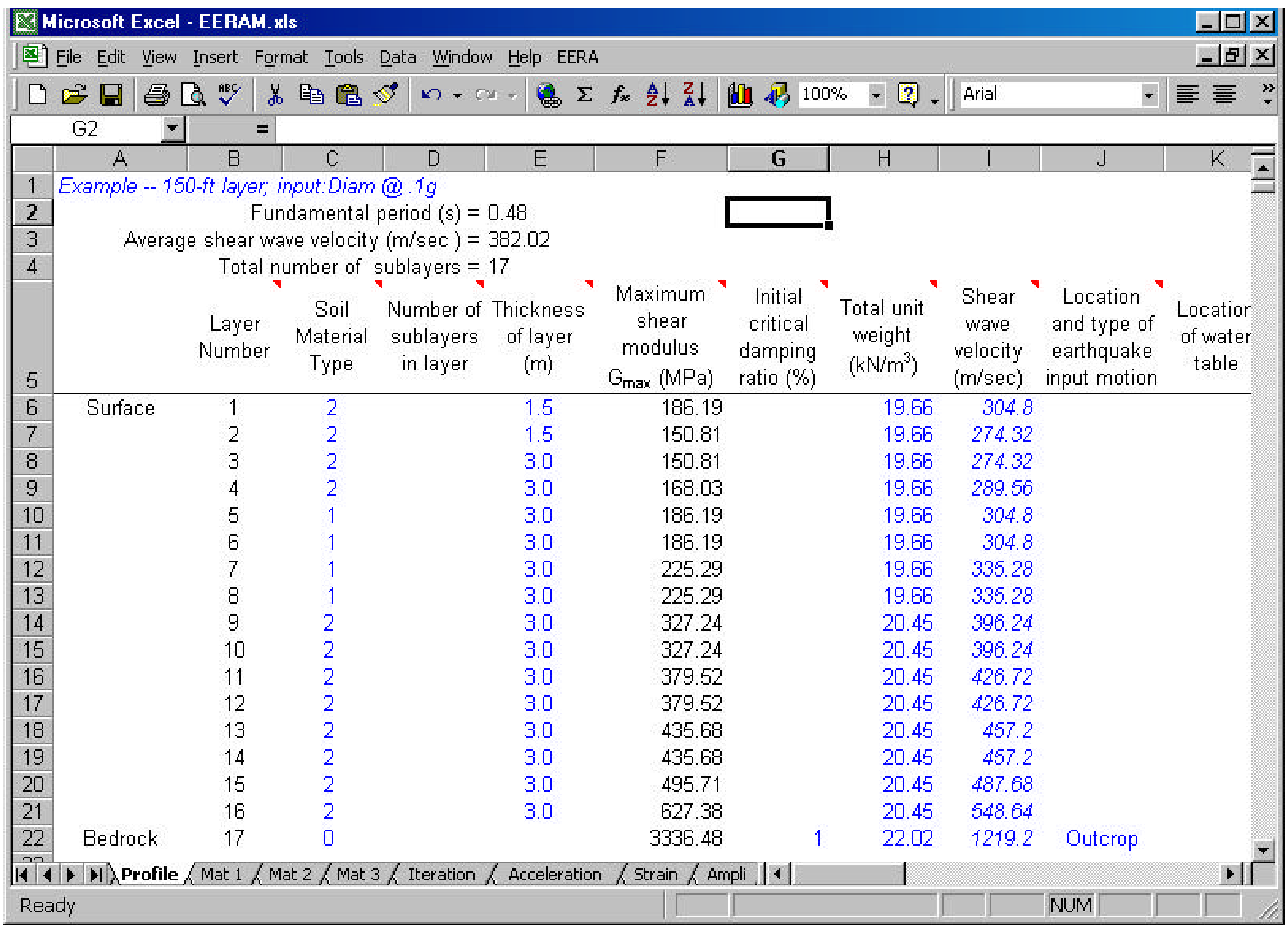
7. APPENDIX A: SAMPLE PROBLEM

The sample problem is identical to the one used for SHAKE91 (Idriss and Sun, 1992). Input data for this sample problem is given in the EXCEL spreadsheets EERA.xls (British units) and EERAM.xls (Metric units).

7.1 Definition of problem

The sample problem is a 150-ft soil profile consisting of clay and sand overlying a half-space. The geometry of the soil layers is defined in Fig. A1. The profiles of shear wave velocity and unit weight are shown in Fig. A2. The three different types of material properties are defined in Figs. A3 to A5. The input motion is specified as an outcrop motion from the acceleration time history recorded at Diamond Heights (EW component) during the 1989 Loma Prieta earthquake. The ground motion is normalized to a target peak acceleration of 0.1 g (Fig. A6).

Figures A1 to A6 shows a snapshot of the actual computer screen. As shown in Figs. A7 and A8 and Table A1, the same results can be displayed and pasted as individual graphs or tables ready for inclusion in technical reports.



Microsoft Excel - EERAM.xls

Example -- 150-ft layer; input:Diam @ .1g

Fundamental period (s) = 0.48

Average shear wave velocity (m/sec) = 382.02

Total number of sublayers = 17

	Layer Number	Soil Material Type	Number of sublayers in layer	Thickness of layer (m)	Maximum shear modulus G_{max} (MPa)	Initial critical damping ratio (%)	Total unit weight (kN/m^3)	Shear wave velocity (m/sec)	Location and type of earthquake input motion	Location of water table
Surface	1	2		1.5	186.19		19.66	304.8		
	2	2		1.5	150.81		19.66	274.32		
	3	2		3.0	150.81		19.66	274.32		
	4	2		3.0	168.03		19.66	289.56		
	5	1		3.0	186.19		19.66	304.8		
	6	1		3.0	186.19		19.66	304.8		
	7	1		3.0	225.29		19.66	335.28		
	8	1		3.0	225.29		19.66	335.28		
	9	2		3.0	327.24		20.45	396.24		
	10	2		3.0	327.24		20.45	396.24		
	11	2		3.0	379.52		20.45	426.72		
	12	2		3.0	379.52		20.45	426.72		
	13	2		3.0	435.68		20.45	457.2		
	14	2		3.0	435.68		20.45	457.2		
	15	2		3.0	495.71		20.45	487.68		
	16	2		3.0	627.38		20.45	548.64		
Bedrock	17	0			3336.48	1	22.02	1219.2	Outcrop	

Profile / Mat 1 / Mat 2 / Mat 3 / Iteration / Acceleration / Strain / Ampli

Figure A1. Definition of soil profile in example problem (EXCEL file EERAM.xls).

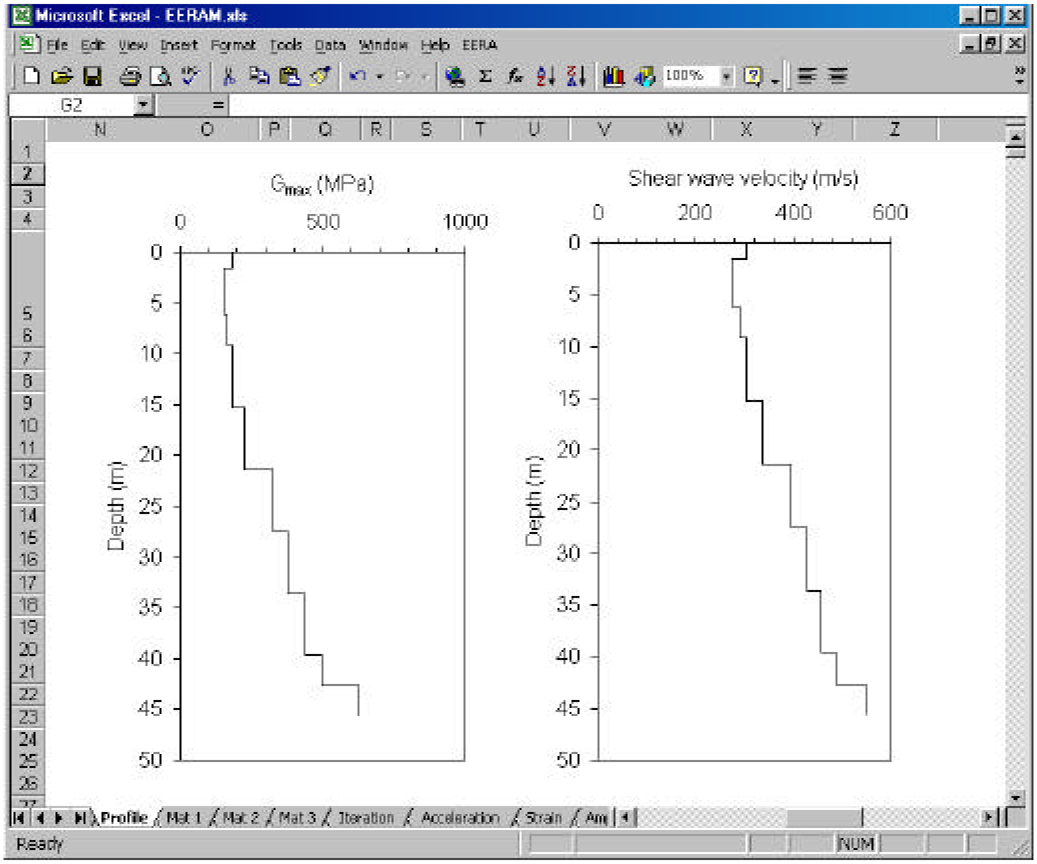


Figure A2. Profiles of shear wave velocity and unit weight of sample problem (EXCEL file EERAM.xls).

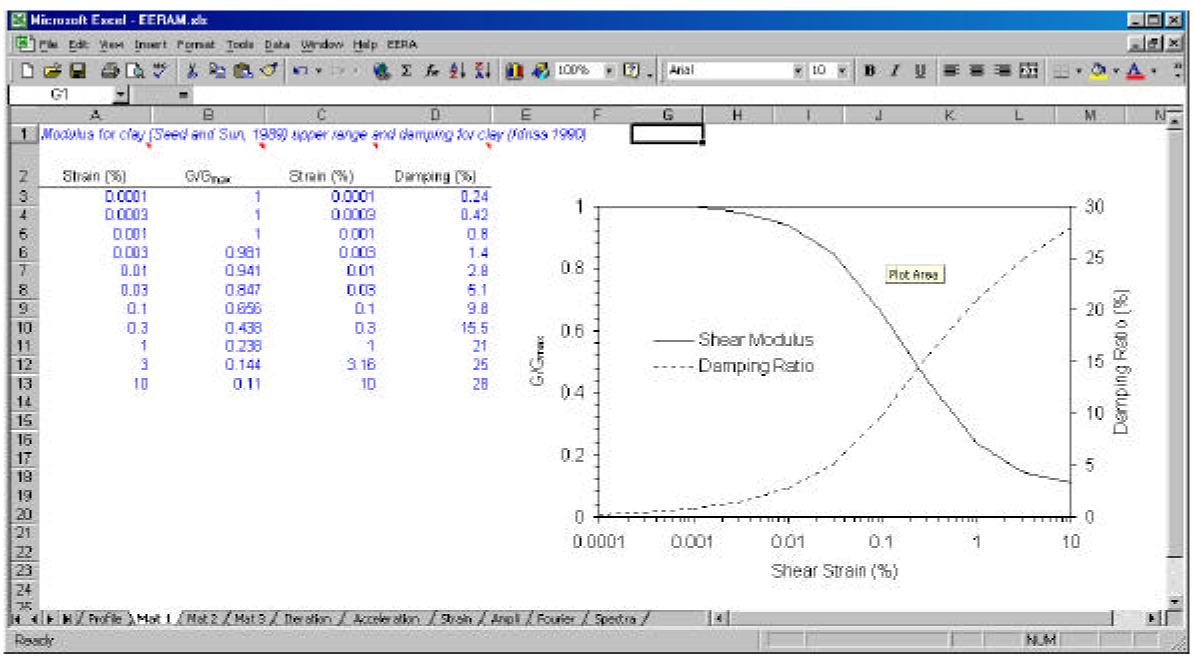


Figure A3. Material properties of material type No.1 (EXCEL file EERAM.xls).

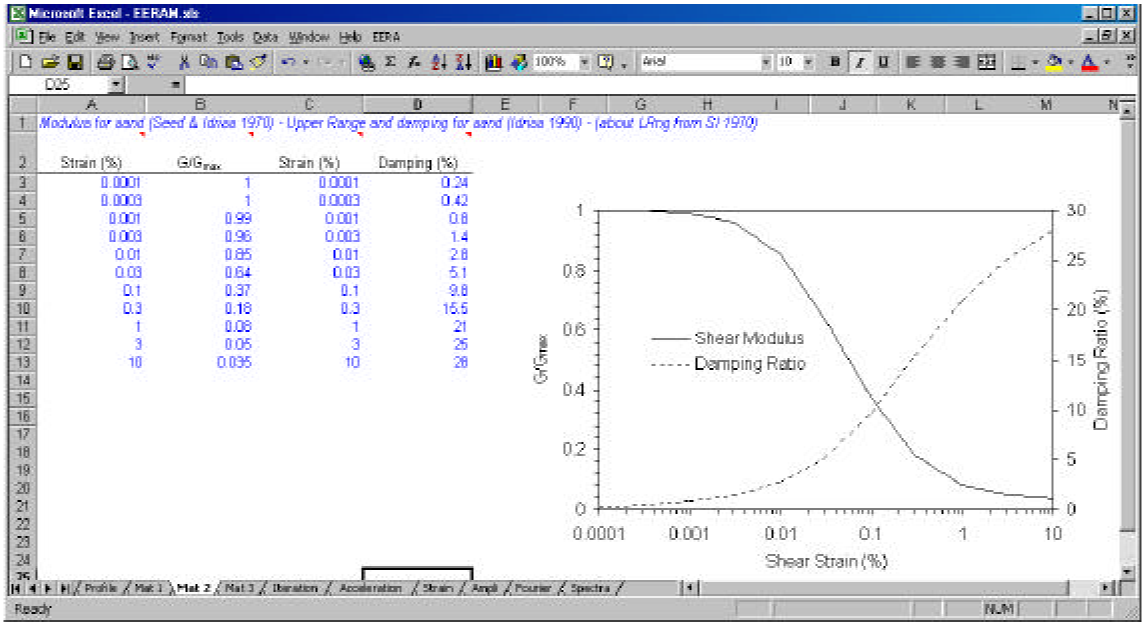


Figure A4. Material properties of material type No.2 (EXCEL file EERAM.xls).

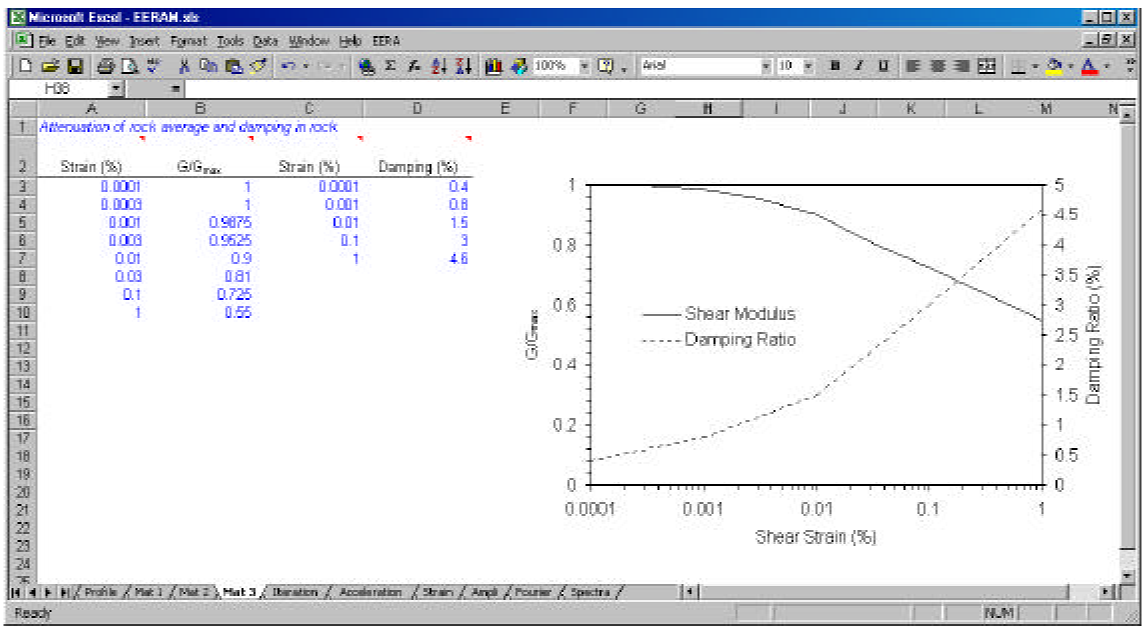


Figure A5. Material properties of material type No.3 (EXCEL file EERAM.xls).

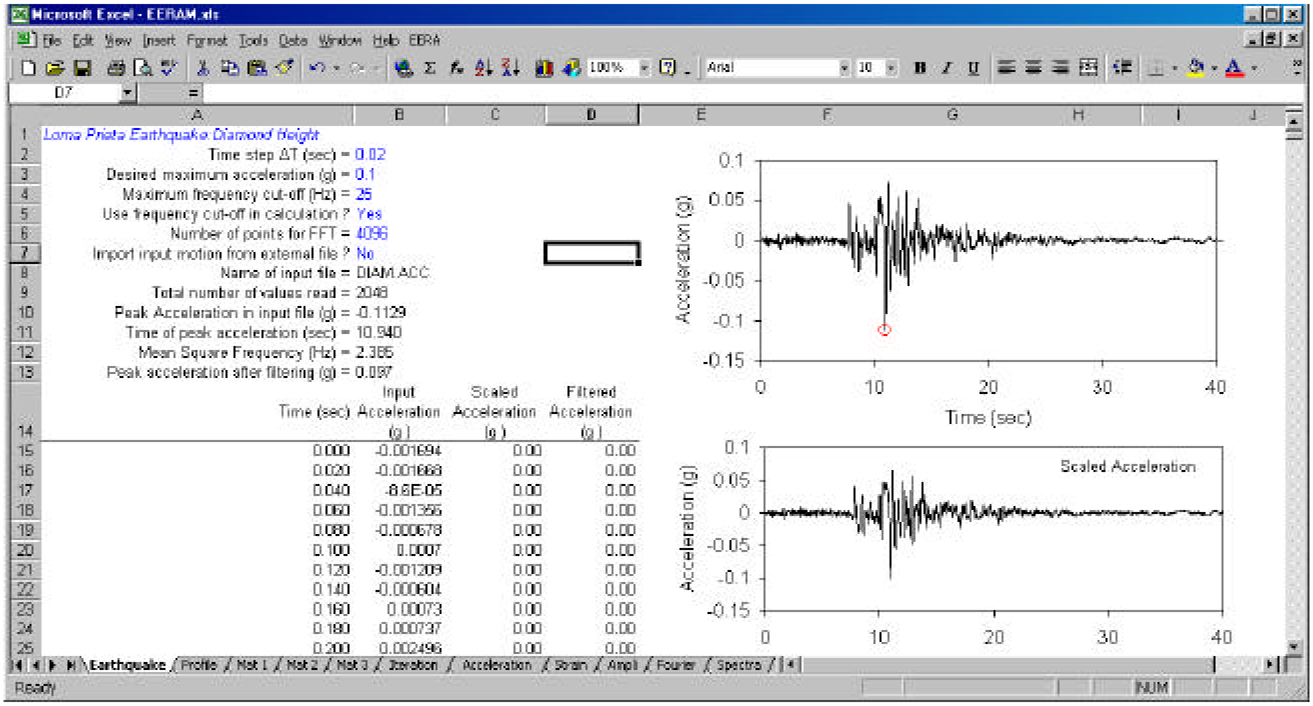


Figure A6. Time history of input ground motion (EXCEL file EERAM.xls).

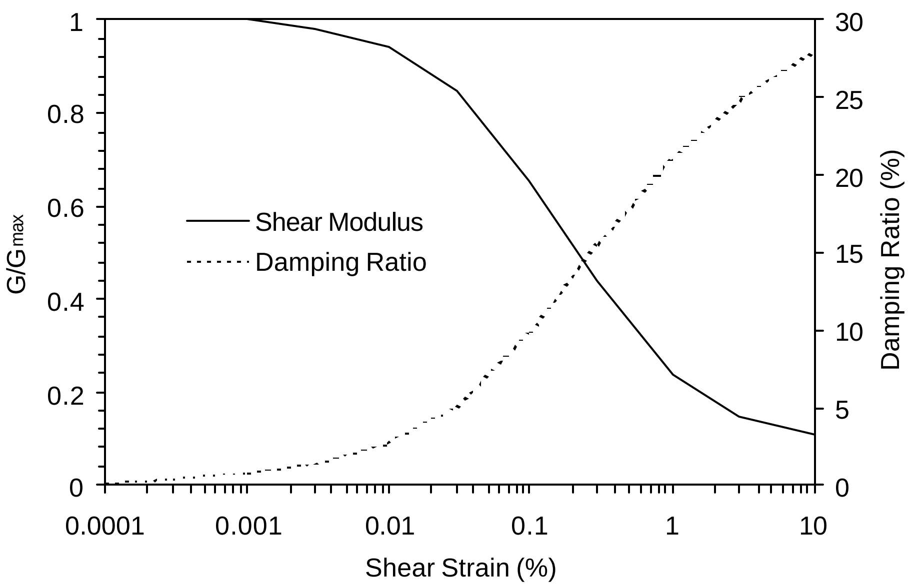


Figure A7. Modulus reduction and damping ratio curves used for sample problem (material No. 1).

Table A1. Values of modulus reduction and damping ratio curves used for sample problem.

Modulus for clay (Seed and Sun, 1989) upper range and damping for clay (Idriss 1990)

Strain (%)	G/G_{max}	Strain (%)	Damping (%)
0.0001	1	0.0001	0.24
0.0003	1	0.0003	0.42
0.001	1	0.001	0.8
0.003	0.981	0.003	1.4
0.01	0.941	0.01	2.8
0.03	0.847	0.03	5.1
0.1	0.656	0.1	9.8
0.3	0.438	0.3	15.5
1	0.238	1	21
3	0.144	3.16	25
10	0.11	10	28

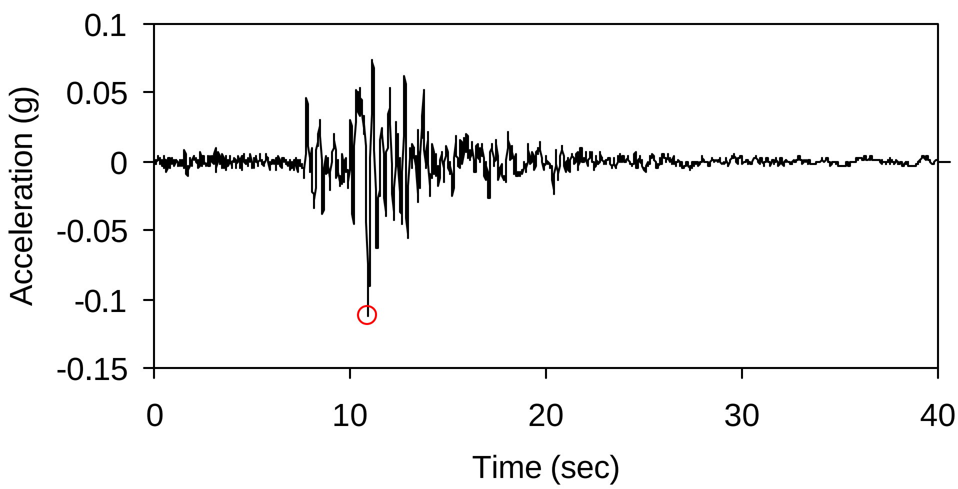


Figure A8. Acceleration time history for Diamond Heights during Loma Prieta earthquake.

7.2 Results

The results of the site response analysis are contained in the EXCEL spreadsheets EERA.xls and EERAM.xls. Some of these results are shown in Figs. A9 to A15.

Figure A9 shows the variation with depth of maximum shear strain, ratio G/G_{max} and damping ratio at each of the calculation iteration step. At the first calculation step, the ratio G/G_{max} is equal to 1, and the damping ratio is constant. After a few calculation steps, the distributions of ratio G/G_{max} and damping ratio converge toward their final values. Figure A10 shows the corresponding variation with depth of maximum shear stress during calculations, and the converged maximum acceleration.

Figure A11 shows the computed time histories of acceleration, relative velocity and relative displacement at the free surface. The relative displacement and velocity are evaluated relative to the bottom of the soil profile. Figure A12 shows the time histories of shear strain, shear stress and energy dissipated per unit volume, and the stress-strain loop computer at sublayer No. 4

Figure A13 shows the computed amplitude of amplification ratio between bottom and free surface. Figure A14 shows the computed amplitude of Fourier amplitude at free surface. Finally, Fig. A15 displays the acceleration response spectrum computed at free surface for a 5% critical damping ratio.

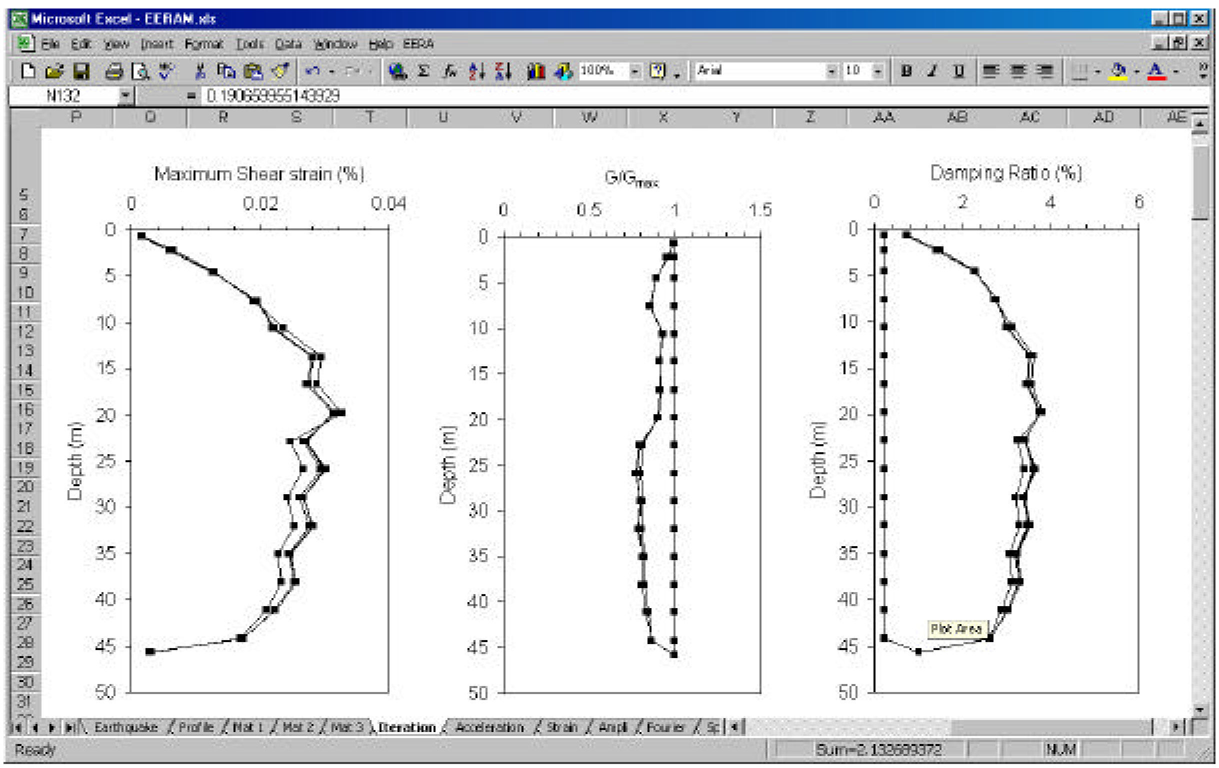


Figure A9. Variation with depth of maximum shear strain, ratio G/G_{max} and damping ratio.

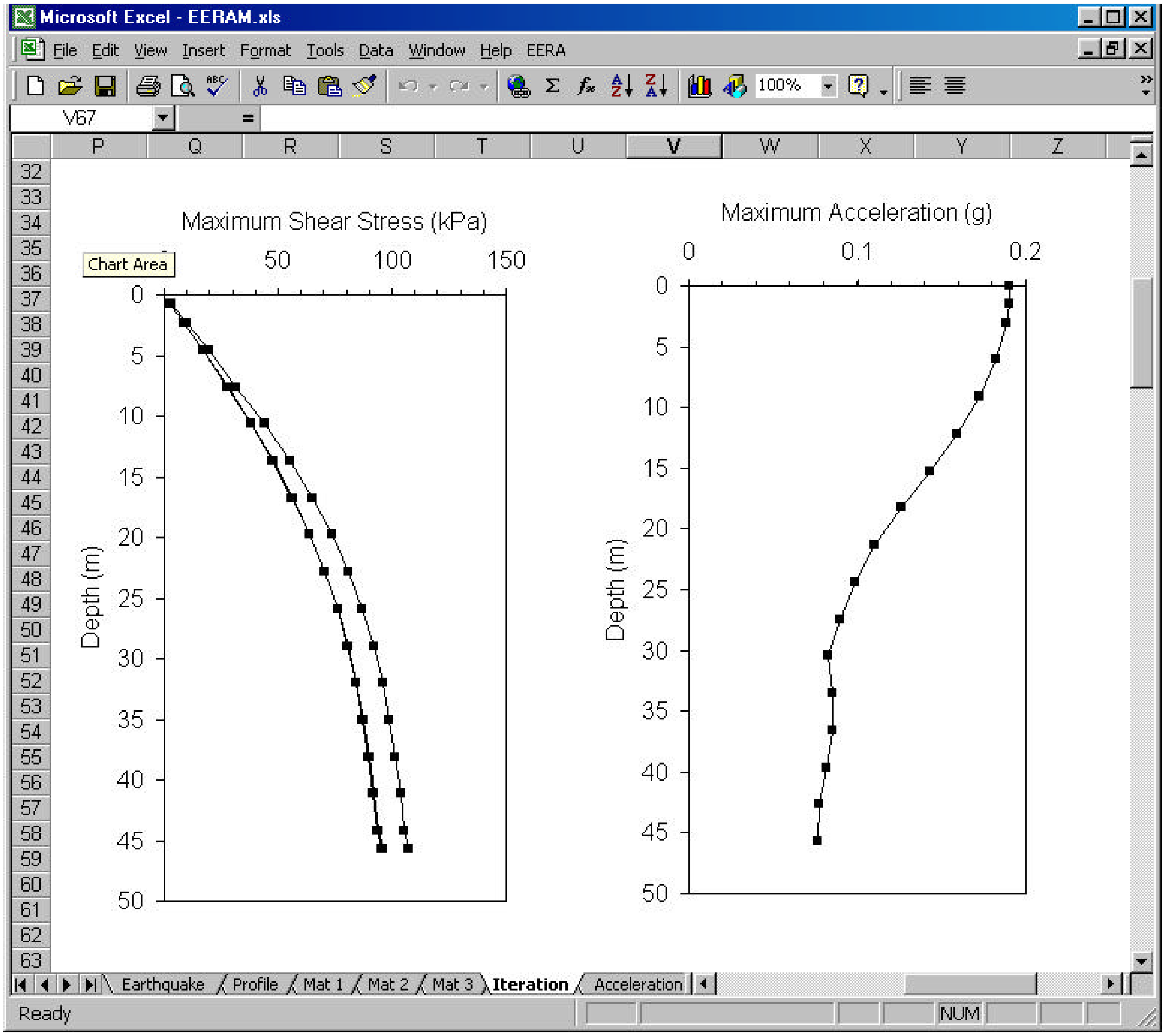


Figure A10. Variation with depth of maximum shear stress during calculation, and converged maximum acceleration.

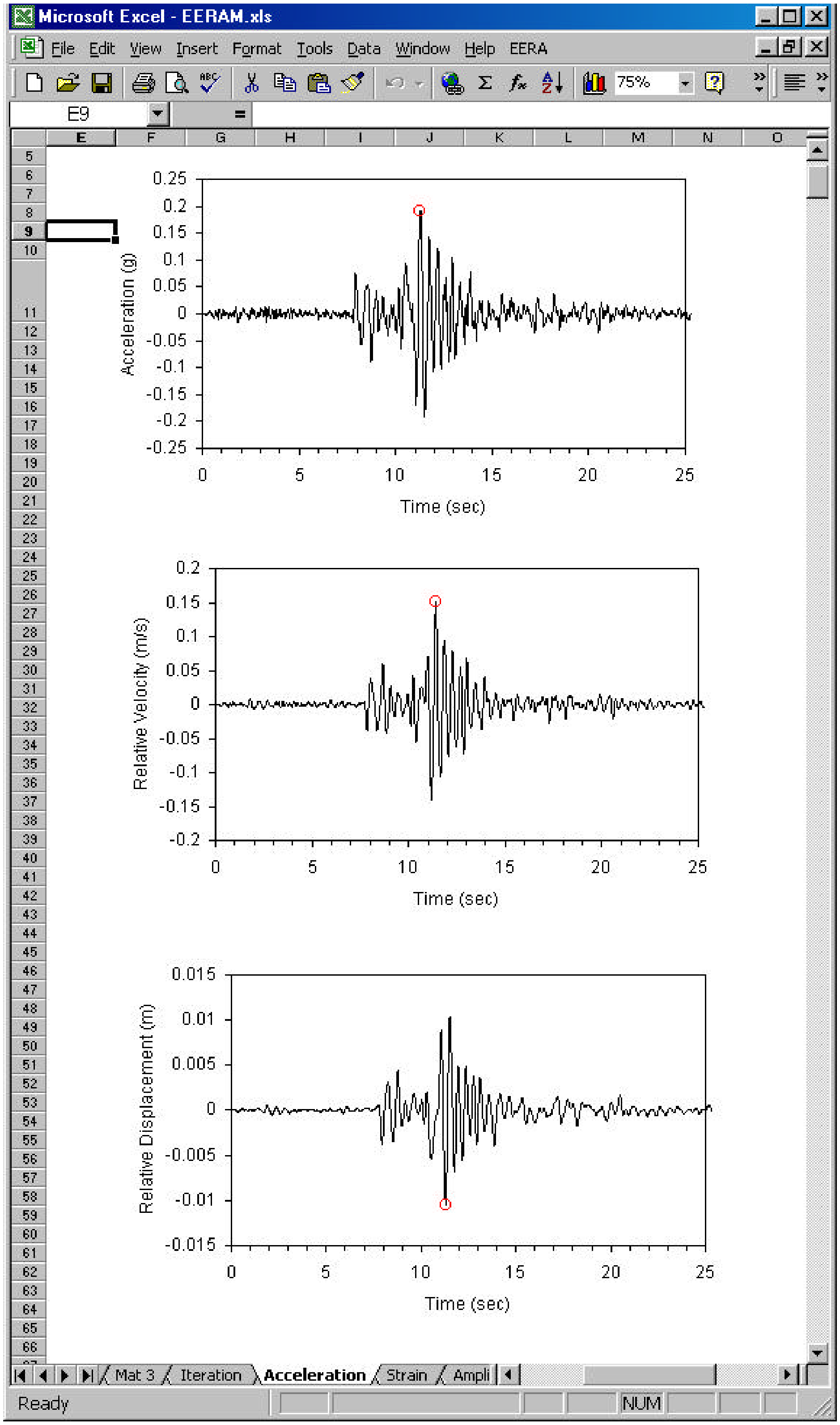


Figure A11. Computed time histories of acceleration, relative velocity and relative displacement at the free surface.

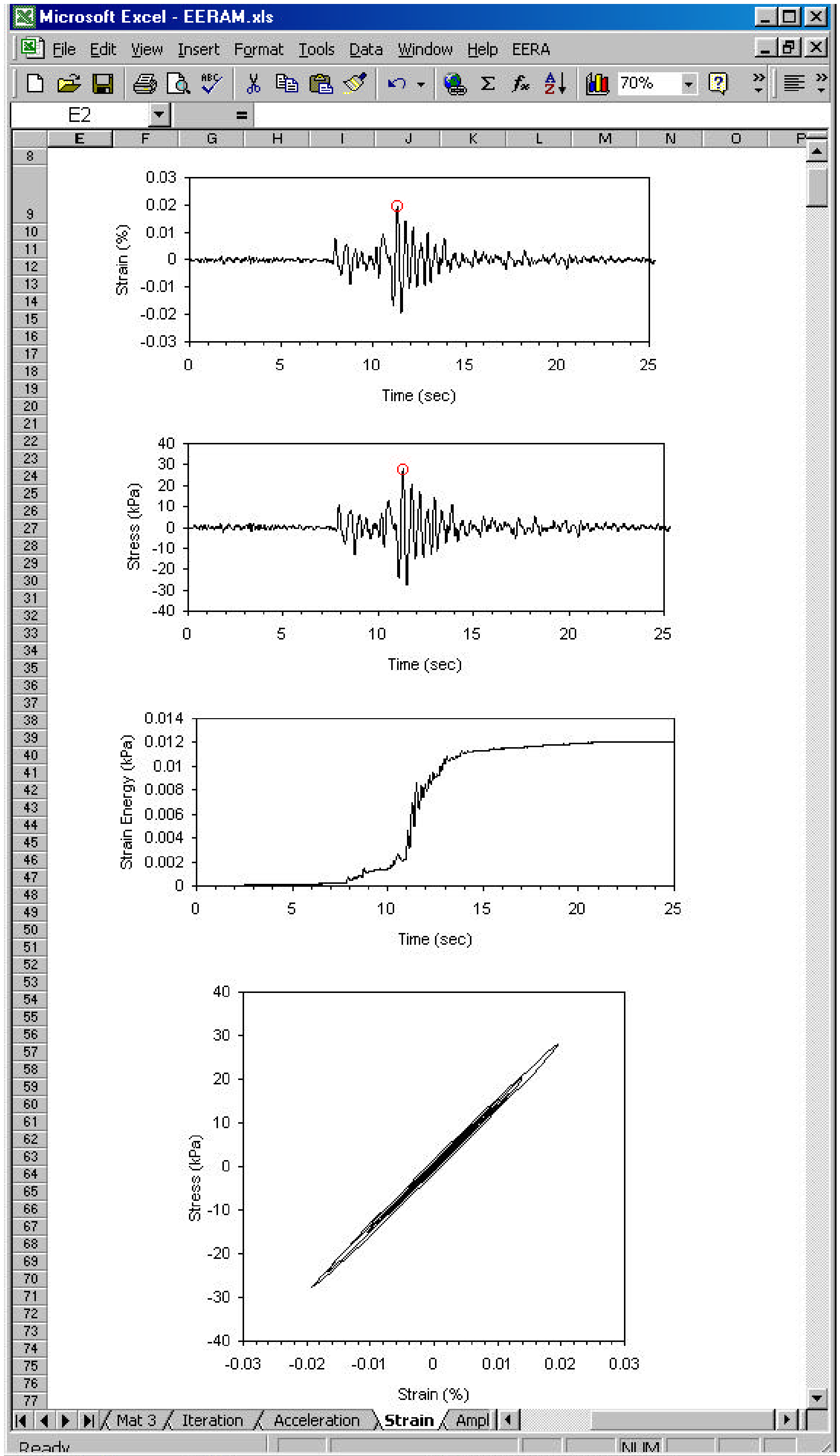


Figure A12. Computed time histories of shear strain, shear stress and energy dissipated per unit volume, and stress-strain loop at sublayer No. 4.

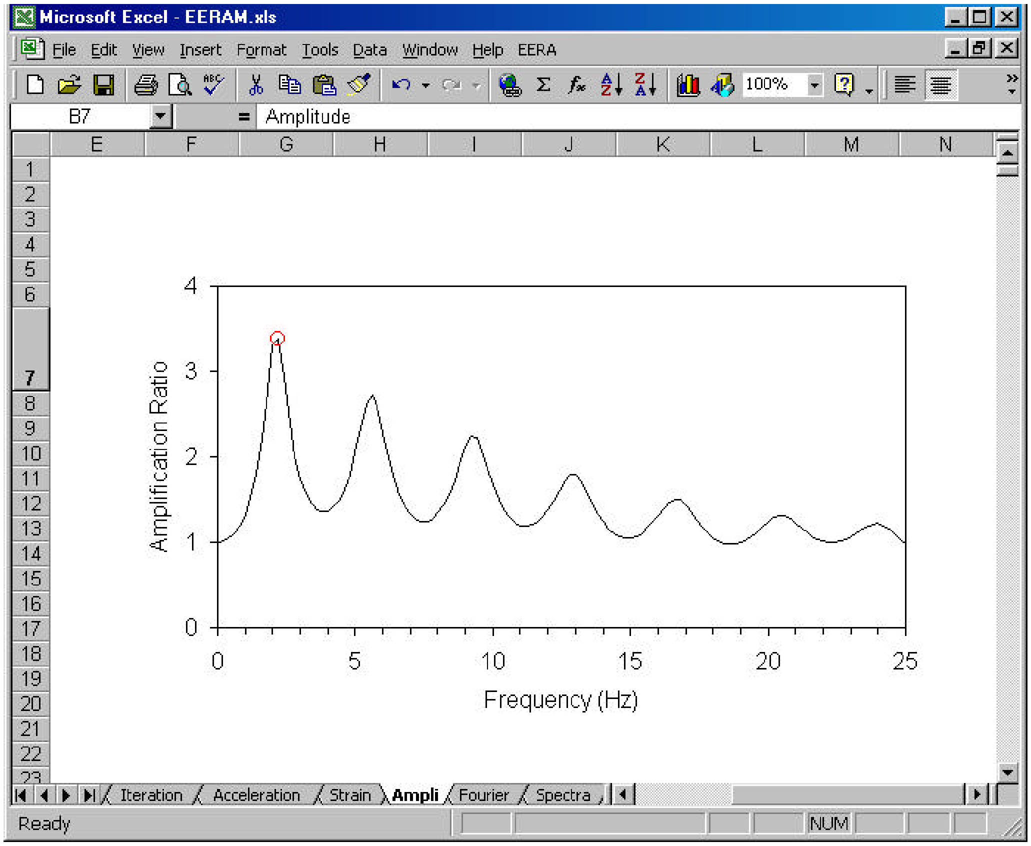


Figure A13. Computed amplitude of amplification ratio between bottom and free surface.

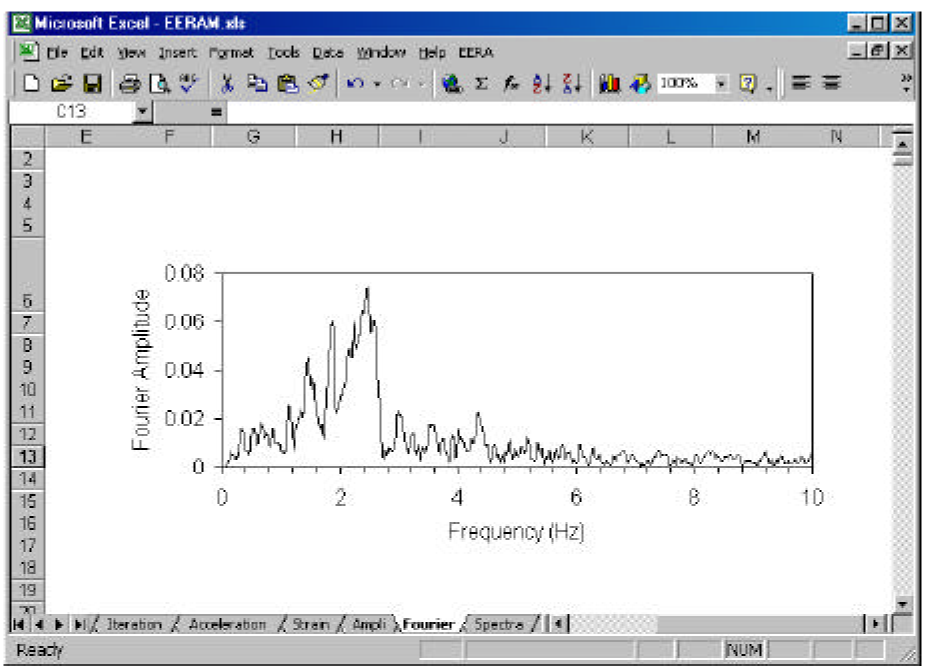


Figure A14. Computed amplitude of Fourier amplitude at free surface.

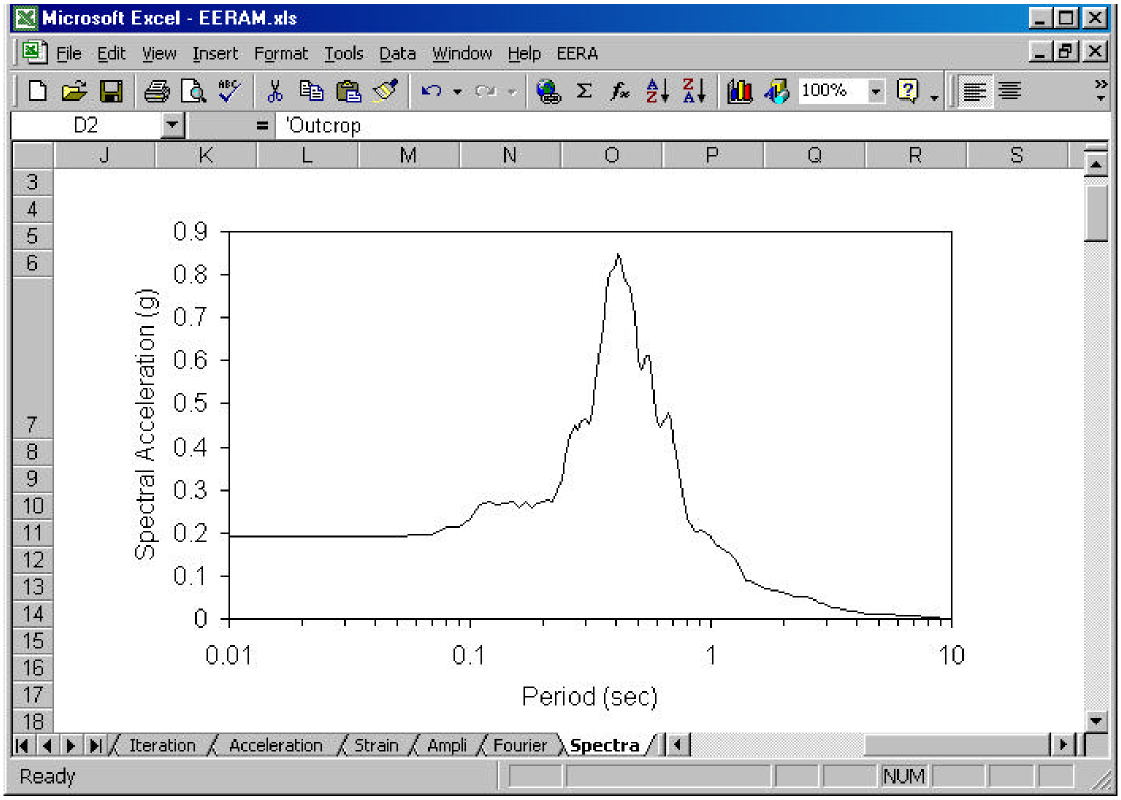
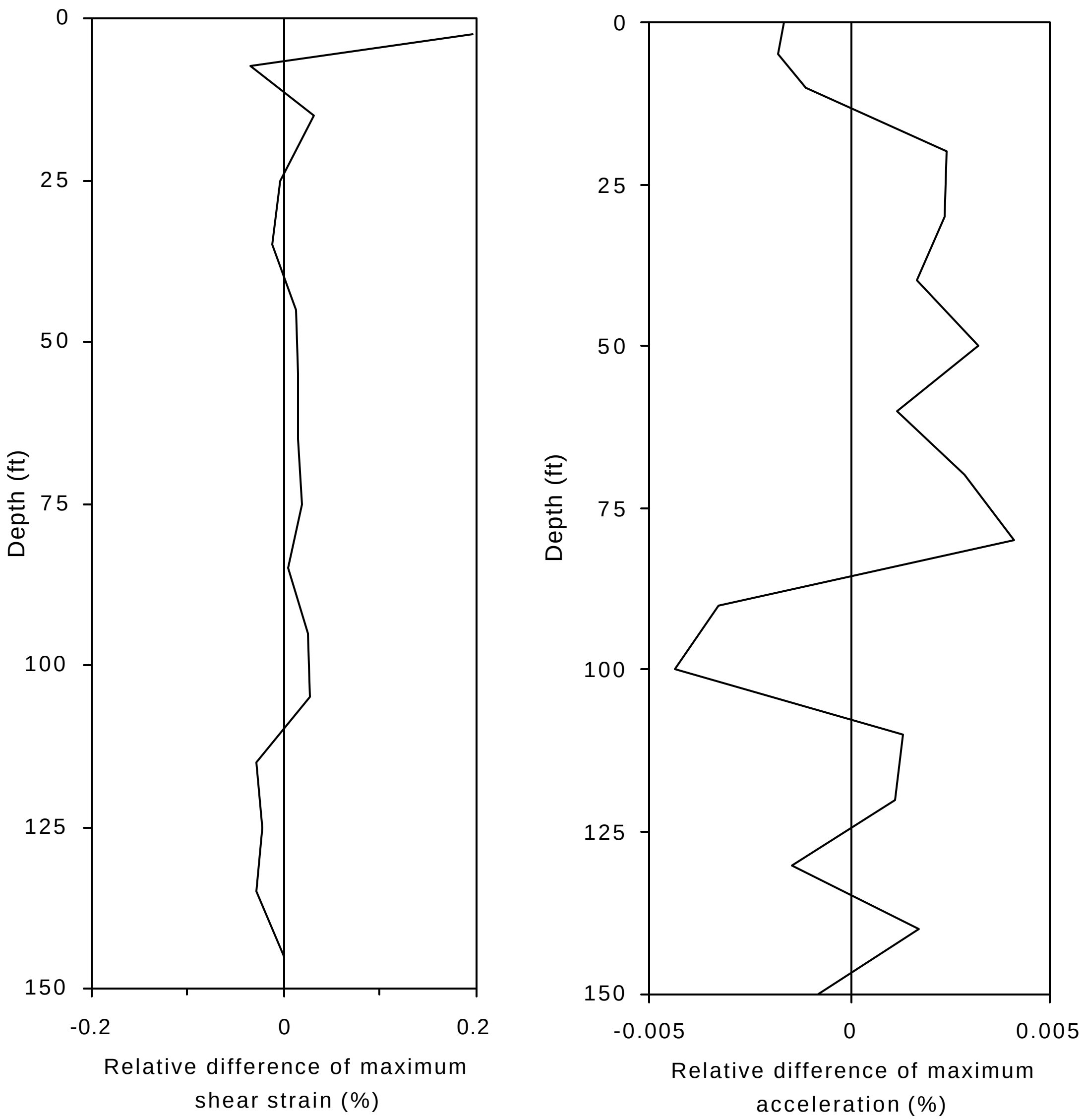


Figure A15. Computed acceleration response spectrum at free surface (5% critical damping ratio).

8. APPENDIX B: COMPARISON OF EERA AND SHAKE91 RESULTS

The results of EERA and SHAKE91 are compared for the particular example problem described in Appendix A. Figures B1 and B2 show the relative difference in EERA results with respect to those of SHAKE91, including maximum shear strain, maximum acceleration and spectral acceleration. As shown in Fig. B1, the increase in the relative difference of shear strain close to the free surface results from a decrease in absolute values of shear strain close to the free surface. As shown in Fig. B2, the increase in the relative difference of spectral acceleration for large period results from the decrease in absolute values of spectral acceleration for large period. In this example, the relative difference was found to be



very negligible for all practical purposes.

Figure B-1 Relative difference of maximum shear strain and maximum acceleration between SHAKE91 and EERA

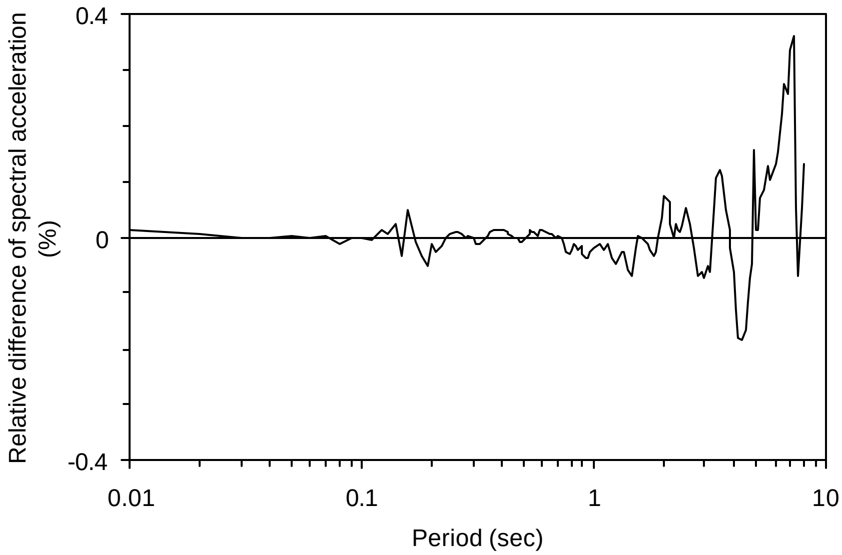


Figure B-2 Relative difference of spectral acceleration calculated by EERA and SHAKE91.